copy 2

APPLIED MECHANICS Reviews

A CRITICAL REVIEW OF THE WORLD LITERATURE IN APPLIED MECHANICS AND RELATED ENGINEERING SCIENCE

REVS. 621-967 VOL. 9, NO. 3 MARCH 1956

GENERAL

Theoretical and Experimental Methods. 92
Mechanics (Dynamics, Statics, Kinematics) 94

MECHANICS OF SOLIDS

Servomechanisms, Governors, Gyroscopics 94
Vibrations, Balancing 95
Wave Motion in Solids, Impact 97
Elasticity Theory 98
Experimental Stress Analysis 99
Rods, Beams, Cables, Machine Elements. 100
Plates, Disks, Shells, Membranes 101
Buckling Problems 103
Joints and Joining Methods 104
Structures 104
Rheology (Plastic, Viscoplastic Flow) 106
Failure, Mechanics of Solid State 107
Material Test Techniques 108
Mechanical Properties of Specific Materials 108
Mechanics of Forming and Cutting 109

MECHANICS OF FLUIDS

Hydraulics; Cavitation; Transport 110
Incompressible Flow: Laminar; Viscous. 111
Compressible Flow, Gas Dynamics 113
Wave Motion in Fluids 115
Turbulence, Boundary Layer, etc. 115
Aerodynamics of Flight; Wind Forces. 117
Aeroelasticity (Flutter, Divergence, etc.). 120
Propellers, Fans, Turbines, Pumps, etc. 121
Flow and Flight Test Techniques 121

HEAT

Thermodynamics 123
Heat and Mass Transfer 125
Combustion 130

MISCELLANEOUS

Acoustics 132
Ballistics, Detonics (Explosions) 134
Soil Mechanics, Seepage 134
Geophysics, Meteorology, Oceanography 135
Lubrication; Bearings; Wear 136
Marine Engineering Problems 136
Biomechanics 136

Books Received, 92
Letters to the Editor, 92

Diffusion in a Fluid in Turbulent Motion, G. K. Batchelor, 89

Published Monthly by
THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
at Easton, Pa., and edited by
Midwest Research Institute with the
co-operation of Linda Hall Library

APPLIED MECHANICS Reviews

Under the Sponsorship of

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS • THE ENGINEERING FOUNDATION • SOUTHWEST RESEARCH INSTITUTE • OFFICE OF NAVAL RESEARCH • AIR RESEARCH AND DEVELOPMENT COMMAND • NATIONAL SCIENCE FOUNDATION

Industrial Subscribers

GENERAL ELECTRIC COMPANY • INTERNATIONAL HARVESTER COMPANY • M. W. KELLOGG COMPANY • GENERAL MOTORS CORPORATION • SHELL DEVELOPMENT COMPANY • WESTINGHOUSE ELECTRIC CORPORATION • WOODWARD GOVERNOR COMPANY

EDITOR Martin Goland

EDITORIAL ADVISERS H. L. Dryden T. von Karman S. Timoshenko

EXECUTIVE EDITOR Stephen Juhasz

ASSOCIATE EDITORS J. C. Shipman Isao Imai J. J. Jaklitsch K. Zarankiewicz H. Norman Abramson

ASSISTANT EDITORS L. Bosch L. Graf J. Demmer S. Lechtman E. Gindorf M. Ingersoll H. K. Wilgus

PUBLICATIONS BUSINESS MANAGER S. A. Tucker

OFFICERS OF ASME J. W. Barker, *President* J. L. Kopf, *Treasurer* C. E. Davies, *Secretary*

AMR MANAGING COMMITTEE R. B. Smith, *Chairman* R. E. Peterson R. D. Mindlin (*ex officio*) H. Vagtborg J. M. Lessells F. J. Weber N. M. Newmark C. Yo

Editorial Office: **APPLIED MECHANICS REVIEWS, Southwest Research Institute, 8500 Culebra Road, San Antonio 6, Texas, U. S. A.**
Subscription and Production Office: **The American Society of Mechanical Engineers, 29 West 39th St., New York 18, N. Y., U. S. A.**

HOW TO OBTAIN COPIES OF ARTICLES INDEXED: Photocopy or microfilm copies of all articles reviewed in this issue will be provided on request. Orders should specify the APPLIED MECHANICS REVIEWS volume and review number; should be addressed to LINDA HALL LIBRARY, 5109 Cherry St., Kansas City 10, Mo., and be accompanied by a remittance to cover costs. Where desirable, photocopies and microfilm may be obtained by teletype, using the number KC334 (call number of LINDA HALL LIBRARY). Photocopy costs are 35c for each page of the article photocopied; minimum charge, $1.25. Microfilm costs include service charge of 50c per article, plus 3c per double page; minimum order, $1.25. (*Applicant assumes responsibility for questions of copyright arising from this copying and use made of copies. Copyright material will not be reproduced beyond recognized "fair use" without consent of copyright owner.*)

APPLIED MECHANICS REVIEWS, March 1956, Vol. 9, No. 3. Published Monthly by The American Society of Mechanical Engineers at 20th and Northampton Streets, Easton, Pa., U. S. A. The editorial office is located at the Southwest Research Institute, San Antonio 6, Texas, U. S. A. Headquarters of ASME, 29 West 39th St., New York 18, N. Y., U. S. A. Cable address: "Dynamic," New York. Price $2.50 per copy, $25.00 a year; to members of ASME $1.00 per copy, $10 a year. Changes of address must be received at Society headquarters seven weeks before they are to be effective on the mailing list. Please send old as well as new address. . . . By-Laws: The Society shall not be responsible for statements or opinions advanced in papers or printed in its publications (B13, Par. 4). . . . Entered as second-class matter, January 11, 1948, at the Post Office at Easton, Pa., under the Act of March 3, 1897. © Copyrighted, 1956, by The American Society of Mechanical Engineers.

APPLIED MECHANICS REVIEWS

VOL. 9, NO. 3 MARTIN GOLAND *Editor* MARCH 1956

DIFFUSION IN A FLUID IN TURBULENT MOTION

G. K. BATCHELOR

TRINITY COLLEGE, CAMBRIDGE, ENGLAND

GENERAL REMARKS

DIFFUSION in turbulent flow is an intractable problem which has been approached from many different points of view. These different viewpoints, which reflect the many contexts in which diffusion plays an important role, each have their uses, but it must be admitted that none is particularly successful and that nothing like a coherent and satisfying picture of the many sides of the problem has been built up. The practical meteorologist or chemical engineer still has great difficulty in making successful predictions of the orders of magnitude of the effects of turbulent diffusion in a new situation. It had better be admitted right away that he is not likely to have less difficulty after reading this article.

In the whole literature of the subject there are comparatively few definitive papers, and most of them are less than twenty years old, so that this short note can be devoted mainly to a description of recent work and of what people are trying to do. A more detailed survey of the present position of research on diffusion in a turbulent fluid has recently been published elsewhere (1).[1]

First let us agree on some obvious preliminaries. Diffusion is essentially a process of statistical redistribution of material particles of the fluid. The substance or quantity that is being "diffused" serves to mark the material particles whose location describes the diffusion, and we are required to determine their subsequent movements in space. Now it is in the nature of diffusion that it is a *random* redistribution of material particles, of which only the average properties are of interest. (It may or may not be clear, in any particular situation, how the average should be taken; for the purposes of general discussion we can avoid the practical difficulties by thinking always of an ensemble average; i.e., of an average over a large number of trials or experiments with identical initial and boundary conditions). The random redistribution that is produced by the thermal motion of the molecules of which the fluid is composed is the familiar process of molecular diffusion. When the bulk motion of the fluid is turbulent, the macroscopic velocity—that is, the mean velocity over the large number of molecules contained in a volume with linear dimensions small compared with a length representative of the boundary—is also random, and a given element of fluid (containing a large number of molecules) moves in different directions in different trials, producing turbulent diffusion. The difference between molecular diffusion and turbulent diffusion is primarily one of scale, the former being characterized by much smaller values of the distances and times over which the velocity correlation coefficient falls to zero.

The whole effect of molecular diffusion is determined, in principle, by the value of the molecular diffusivity appropriate to the quantity that serves to mark the fluid. When this diffusivity is zero—as it is, approximately, for ionic salts dissolved in water, for example—we are left (in turbulent flow) with pure turbulent diffusion. Provided the quantity that is being diffused has no effect on the dynamical properties of the fluid (as we shall assume, although it should be noted that this excludes the important case of diffusion of heat with appreciable temperature differences, in which buoyancy forces play a significant part), its precise character is irrelevant to pure turbulent diffusion.

If the motion of the fluid were nonturbulent, the action of the motion on the position of the marked fluid particles would normally be described by the term "convective transport." This is anything but a simple process, as anyone will know who has put a drop of ink in a glass of water that has been stirred gently. Very little study of the paths of marked fluid particles in nonturbulent motions has been made, and it is probable that many general results are waiting to be found. It is characteristic of most steady motions of a fluid that the separation of fluid particles on neighboring streamlines increases indefinitely, so that a finite connected volume of marked fluid will ultimately be drawn out into a sheet of ever-decreasing thickness. It is then necessary to take account of the action of molecular diffusion on the spatial distribution of the property used to mark the fluid particles, because, however small the appropriate molecular diffusivity may be, the molecular transfer in directions normal to the thin sheet ultimately becomes appreciable, as Ekart (2) has pointed out. This complicates the problem considerably, because it involves a mixture of the effect of steady convection, which is better thought of in terms of a Lagrangian representation of the motion, and the effect of molecular diffusion, which depends on instantaneous spatial relationships and is better handled by a Eulerian reference system. Taylor (3) has shown that in the simple case of Poiseuille flow the tendency for the convection to extend the region of marked fluid in the longitudinal or flow direction is opposed by the molecular diffusion of marked fluid across the streamlines, and that when the lateral gradient of concentration of marked fluid is everywhere small, the combined process can be described in terms of a longitudinal diffusivity (which is inversely proportional to the molecular diffusivity).

[1] Numbers in parentheses indicate References at end of paper.

When the motion of the fluid is turbulent, there are three separate effects to contend with. There is first the convection of marked fluid along the streamlines of the mean flow, with accompanying distortion of the distribution of marked fluid if the mean flow velocity is not uniform. There is also the convection due to turbulent fluctuations in velocity of the fluid; this convective transfer is random, only its statistical properties being capable of prediction (in practice), and is therefore better regarded as a "diffusion" of marked fluid. The third process is molecular diffusion of marked fluid, and although the molecular diffusivity will normally be much smaller than the effective diffusivity of the turbulent motion, molecular diffusion cannot be neglected in any situation in which the gradients of concentration of marked fluid are made large by convective distortion (arising from either the mean or fluctuating motions). This latter point has not generally been recognized, but Townsend (4) has shown how molecular diffusion has a significant effect on the spread of heat behind a line source placed in a uniform stream carrying homogeneous turbulence.

Problems of turbulent diffusion are consequently rather complex and many-sided, and there are considerable variations—of a fundamental, as well as of a practical character—between one particular problem and another. Perhaps the most important analytical task at the moment is to *classify* the various kinds of situations that can arise, and to perceive basic features that are common to groups of them. At the present time, we are apt to talk, for instance, of the problem of diffusion in turbulent shear flow; this is not one problem, but many, and an improved understanding of this kind of diffusion is likely to be obtained only when we have succeeded in recognizing a number of more particular cases of turbulent shear flow which isolate, so far as is possible, the various processes that are present in the general case. The remainder of this article will describe some of the particular features of diffusion in turbulent fluid that have already been noticed and are being given some attention.

HOMOGENEOUS TURBULENCE

When the mean velocity and the statistical properties of the turbulent motion are uniform, as they are (approximately) in a wind-tunnel stream, there is no distortion of the region of marked fluid by the mean flow. Carrying the simplification of conditions a step further, we may enquire into the combined effect of turbulent and molecular diffusion on a one-dimensional distribution of mean concentration of marked fluid ($=\bar{C}$, say). Such a one-dimensional distribution might be produced by suddenly releasing heat uniformly over a plane in the fluid (temperature at any point then being a measure of $\bar{C}$), as was indeed done experimentally by Schubauer (5) and Simmons (see Taylor (6)) and by many other people since 1935. The marked (or hot) fluid is here initially in the form of a thin plane sheet, which is subsequently thickened by molecular transfer and distorted by turbulent convection. The result of molecular diffusion alone is to make the thickness of the sheet increase as $t^{1/2}$ (more precisely, this form of dependence on t holds at times t large enough for many molecular collisions to have occurred), and an analogous result is valid for pure turbulent diffusion. In a pioneering paper, Taylor (7) pointed out that turbulent diffusion alone would cause the width of the distribution of mean temperature to increase first as t (while the velocities of fluid particles are still approximately as they were at the initial instant), and ultimately as $t^{1/2}$ (when the particle velocities have lost correlation with their initial values; i.e., after many "eddy collisions" have been made).

The experimental observations had seemed at first sight to support the commonly accepted view that the total diffusion of the hot sheet was a linear combination of the effects of molecular and turbulent diffusion separately, and it was thought that the last word had been said on this particular case. However, Townsend (4) has recently produced strong arguments against this view; it seems, according to this later work, that the increased molecular diffusion arising from the distortions in the sheet is significant, irrespective of the value of the Reynolds number. The interpretation of these arguments is that the effect of increasing the Reynolds number of the turbulence (the Prandtl number remaining constant) is to make the sheet thinner at any given value of t (expressed nondimensionally, using a length and a velocity representative of the turbulence as a whole), and also to increase the number of undulations per unit area produced by turbulent convection, the net effect being to make finite the contribution to the width of the distribution of mean temperature from molecular transfer, however large the Reynolds number. The exact value of this contribution to the width of the temperature distribution has not yet been determined, except at very small values of t.

This demonstration that the contribution from molecular diffusion is still significant in the limit of infinite Reynolds number, and takes a limiting value that depends on the ratio of the kinematic viscosity to the molecular diffusivity of the marked fluid, is a fundamental advance that raises many new and interesting questions. Whether it is equally true in all other cases of turbulent diffusion is not yet known.

CASES OF SHEAR FLOW WITH PARTIAL HOMOGENEITY

The feature of homogeneous turbulence that renders the analysis of diffusion problems relatively simple is that the velocity of a material fluid element is a stationary random function of time (or can be made so, by a mathematical transformation, if the turbulence is decaying). This feature is retained in cases of turbulent shear flow in which the fluid is flowing through a long straight tube, of arbitrary cross section. It can be shown in such cases that the mean velocity of a material fluid element (the mean being taken either over many trials or over a long interval of time) is equal to the discharge velocity, or mean volume flux across any section divided by the cross-sectional area. Consequently the displacement of a material element in a given time t consists of a mean Ut, where U is the discharge velocity, together with a fluctuation whose root-mean-square is at first proportional to t and ultimately becomes proportional to $t^{1/2}$. This latter result has been confirmed by Taylor (8), who injected salt solution into water flowing through a circular tube and observed the spreading of the salt far downstream where the various fluid elements containing salt can be regarded as having statistically independent velocities, and by Batchelor, Binnie, and Phillips (9), who timed the passage of small wax spheres past various stations along a circular tube containing water. In this latter case, it was possible to allow for the effect of the finite size of the spheres (which were prevented from being influenced by the region of very low velocity close to the wall) on their mean velocity and on the spread of times. It will be noticed that the spread (over many trials) of times of travel between fixed stations is a diminishing fraction of the mean time of travel as the latter increases, so that observation of the time of travel of a single particle (preferably a large sphere, since the spread of times decreases as the particle radius increases) over a sufficiently long length of pipe provides a convenient method of determining the discharge velocity.

At these large intervals of time (large in the sense that the fluid elements have lost correlation with their initial velocity), the probability distribution of the distance traveled down the pipe by a fluid element in a fixed time is Gaussian, or equivalently, the mean concentration of marked fluid satisfies a heat

conduction equation, the effective diffusivity being related to the spread of times as measured in either of the above two ways. (Taylor (8) has shown that the spread can also be computed by making use of the Reynolds analogy between lateral turbulent transfer of momentum and of marked fluid, together with empirical information about the transfer of momentum.) This differential equation can be used to solve a number of practical problems of longitudinal diffusion; an example is the determination of the mixing which occurs when the fluid supplied at the inlet of a pipe is suddenly replaced by a different fluid.

DIFFUSION FROM A MAINTAINED SOURCE IN SHEAR FLOW

If a steady source of, say, heat is placed in a turbulent shearing motion, the wake of hot fluid extending downstream from the source is affected by all three of the processes mentioned earlier. The relation of the distribution of mean temperature over a plane downstream from the source to the statistics of the motion of fluid elements passing close to the source is here more complicated, and there is insufficient space in this article for a proper account of the situation. Many of the complications arise from the fact that the fluctuations in velocity are usually not a negligible fraction of the mean velocity in turbulent shear flows (unlike the case of turbulence in a uniform stream in a wind tunnel). Measurements of this kind have been made by Skramstad (10) in a boundary layer, by Corrsin and Uberoi (11) in a round jet, and by Hinze and van der Hegge Zijnen (12) in a two-dimensional jet; and a discussion of the (rather limited) information that can be derived from these observations has been given elsewhere (1).

RELATIVE DIFFUSION OF MARKED FLUID

Another kind of problem of turbulent diffusion, and one that is especially of interest to meteorologists, concerns the way in which elements of marked fluid diffuse relative to each other. When two elements are so far apart that their simultaneous velocities are statistically independent, their relative diffusion is simply obtained from a knowledge of the diffusion of each of them, separately, relative to a fixed point. When they are not so far apart, the fact that the two elements are simultaneously influenced by the same "eddy" must be taken into account. The relative velocity of two fluid elements whose mean distance apart is r is influenced principally by those eddies (or Fourier components of the turbulent velocity field) that have a length scale of order no larger than r, and as the elements wander further apart the effect of a wider range of eddy sizes leads to an increase in the mean-square relative velocity. This increase in turn leads to a more rapid (statistical) wandering apart. Thus relative diffusion of fluid elements that initially are close together has an accelerating character—as was first recognized by Richardson (13)—which can be given explicit analytical form with the help of the universal equilibrium theory concerning the small-scale components of the turbulence (14). Perhaps the most important theoretical result here is that the temporal rate-of-change of the mean-square separation of two fluid elements is proportional to the square of the time, provided that (a) the time interval is large enough for the statistical influence of the given initial separation to have been lost, and (b) the separation is small compared with the size of the energy-containing eddies and large compared with the size of eddies affected by viscosity.

An interesting and significant special case of relative diffusion is obtained by allowing the two fluid elements to be so close together that all spatial derivatives of the velocity in the fluid are uniform over the distance between the elements. More generally, we can enquire into the statistical properties of the shape of a volume of marked fluid that initially is spherical, with a radius so small that at all the later times to be considered the volume is ellipsoidal. It is not difficult to see that the means of the three principal semiaxes of the ellipsoid will vary exponentially with time, the three exponents being ξt, ηt, $(-\xi-\eta)t$, say, where $\xi > 0$ and $\xi > \eta$; and that as a consequence all material diameters of the original sphere tend to become parallel to each other, and likewise all material diametral planes. Some observations by Townsend (15) of the temporal variation of the maximum temperature of a single hot element of fluid in homogeneous turbulence suggest that there is little rotation, relative to the fluid, of the principal axes of the rate-of-strain experienced by the element, or, equivalently, that the principal axes of the rate-of-strain and the principal axes of the ellipsoid remain nearly coincident (which, it will be noted, leads to the maximum rate of cooling of the hot element). There is also some evidence to show that $\eta > 0$, so that the ellipsoid ultimately has two semiaxes that are large compared with the radius of the original sphere.

These results about the statistical properties of the rate of strain experienced by a material element of fluid have interesting implications for the way in which turbulent motion acts on certain vector fields associated with the fluid. A vector field *grad* θ, where θ represents any passive local physical property, like concentration of coloring matter, is changed in such a way that its magnitude increases in proportion to the area of a local material surface element, whereas a vector field **F** which satisfies conservation of flux, like vorticity or magnetic field strength in a conducting fluid, changes in such a way that its magnitude increases in proportion to the length of a local material line element. Thus a connection between relative diffusion and the action of turbulence on physical properties of the fluid can be made. A more complete account is given in the survey already quoted.

REFERENCES

1 Batchelor, G. K., and Townsend, A. A., Turbulent diffusion. Article in *Surveys in mechanics*, Cambridge University Press, 1956.

2 Ekart, C., An analysis of the stirring and mixing processes in incompressible fluids, *J. Mar. Res.* **7**, p. 265, 1948.

3 Taylor, G. I., Dispersion of soluble matter in solvent flowing slowly through a tube, *Proc. roy. Soc. Lond.* (*A*) **219**, p. 186, 1953.

4 Townsend, A. A., The diffusion behind a line source in homogeneous turbulence, *Proc. roy. Soc. Lond.* (*A*) **224**, p. 487, 1954.

5 Schubauer, G. B., A turbulence indicator utilizing the diffusion of heat, NACA Technical Note 524, 1935.

6 Taylor, G. I., Statistical theory of turbulence. IV. Diffusion in a turbulent air stream, *Proc. roy. Soc. Lond.* (*A*) **151**, p. 465, 1935.

7 Taylor, G. I., Diffusion by continuous movements, *Proc. Lond. math. Soc.* **20**, p. 196, 1921.

8 Taylor, G. I., The dispersion of matter in turbulent flow through a pipe, *Proc. roy. Soc. Lond.* (*A*) **223**, p. 446, 1954. AMR **7**, Rev. 3285.

9 Batchelor, G. K., Binnie, A. M., and Phillips, O. M., The mean velocity of discrete particles in turbulent flow in a pipe, *Proc. phys. Soc. Lond.* (*B*) **68**, p. 1095, 1955.

10 Dryden, H., Turbulence and diffusion, *Indust. Engng. Chem.* **31**, p. 416, 1939.

11 Corrsin, S., and Uberoi, M. S., Spectra and diffusion in a round turbulent jet, NACA Rep. 1040, 21 pp., 1951. AMR **5**, Rev. 3476.

12 Hinze, J. O., and van der Hegge Zijnen, B. G., Local transfer of heat in anisotropic turbulence, Instn. mech. Engng., General discussion on heat transfer, 1951, p. 188.

13 Richardson, L. F., Atmospheric diffusion shown on a distance-neighbor graph, *Proc. roy. Soc. Lond.* (*A*) **110**, p. 709, 1926.

14 Batchelor, G. K., Diffusion in a field of homogeneous turbulence. II. The relative motion of particles, *Proc. Camb. phil. Soc.* **48**, p. 345, 1952. AMR **5**, Rev. 2431.

15 Townsend, A. A., The diffusion of heat spots in isotropic turbulence, *Proc. roy. Soc. Lond.* (*A*) **209**, p. 418, 1951. AMR **5**, Rev. 1498.

Books Received for Review

DEICH, M. E., Tekhnicheskaia gazodinamika, Moscow, Leningrad, Gosudarstvennoe Znergeticheskoe Izdatelvstvo, 1953, 544 pp. $2.50.

FISCHER, F. A., Fundamentals of electroacoustics [Grundzüge der Elektroakustik], Berlin, Fachverlag Schiele & Schön, 1950 (translated by Ehrlich, S., and Pordes, F.), New York, London, Interscience Publishers, Inc., 1955, xii + 186 pp. $6.

GÖBEL, E. F., Berechnung und Gestaltung von Gummifedern, 2nd ed. (Konstruktionsbücher 7), Berlin, Göttingen, Heidelberg, Springer-Verlag, 1955, v + 86 pp. DM 9 (paperbound).

GULDAN, R., Die Cross-Methode und ihre praktische Anwendung, Wien, Springer-Verlag, 1955, xix + 472 pp. $17.15.

JOHN, F., Plane waves and spherical means applied to partial differential equations (Interscience Tracts in Pure and Applied Mathematics, 2), New York, London, Interscience Publishers, Inc., 1955, viii + 172 pp. $4.50.

KANI, G., Spannbeton in Entwurf und Ausführung, Stuttgart, Verlag Konrad Wittwer, 1955, xii + 573 pp. DM 45.

LEWIS, B., PEASE, R. N., AND TAYLOR, H. S., editors, Combustion processes (High Speed Aerodynamics and Jet Propulsion, II), Princeton, New Jersey, Princeton University Press, 1956, xv + 662 pp. $12.50.

LIN, C. C., The theory of hydrodynamic stability (Cambridge Monographs on Mechanics and Applied Mathematics), Cambridge University Press, 1955, xi + 155 pp. $4.25.

LIN, T. Y., Design of prestressed concrete structures, New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd., 1955, ix + 456 pp. $11.50.

PRESTRESSED CONCRETE (ACI Bibliography no. 1), 2nd ed., Detroit, American Concrete Institute, 1955, 98 pp. + 5 patents. $2.

RADOK, J. R. M., Die stabilität der versteiften platten und schalen, Groningen, Djakarta, P. Noordhoff N. V., 47 pp. $0.75 (paperbound).

SCHULZ, H., Die Pumpen, Arbeitweise, Berechnung, Konstruktion, 9th ed., Berlin, Gottingen, Heidelberg, Springer-Verlag, 1955, vii + 188 pp. DM 16.50.

WALKER, W. F., Fundamentals of press tool design, New York, Philosophical Library, Inc., 1955, 152 pp. $4.75.

Letters to the Editor

621. Re feature article, AMR 8 (September 1955): Kézdi, A., Soil mechanics.

Since writing my feature article, it has been called to my attention that the theory of three-dimensional consolidation has been successfully dealt with in the papers of M. A. Biot also. Unfortunately, his original papers [*J. appl. Phys.*, 1941–1942] were not available to me and I knew of them only through references made by others. A. Kézdi, Hungary

622. Re AMR 8, Rev. 3652 (December 1955): Eubanks, R. A., Muster, D., and Volterra, E., On the attenuation of a sinusoidal pulse in a cylindrical specimen held between elastic bars.

As the senior coauthor of this paper, I would like to make the following remarks concerning the review by Professor W. H. Hoppmann, II. I agree with the reviewer's statement that there is a present need for more experimental research on the response of real materials to impact loadings to give a better understanding of their dynamic characteristics. The senior coauthor has been doing experimental research work along these lines since the year 1943. But Professor Hoppmann seems to have misunderstood the object of the present paper, which is a purely theoretical one. Solutions of the problems of one-dimensional wave propagation through elastoviscous and hereditary media have been given previously by I. N. Zverev and H. Charles, respectively. The Zverev solution was given in the form of a double integral, while the Charles solution was given as a fourfold iterated integral. Instead, our solutions in both cases considered were given under the forms of single integrals whose numerical evaluations are much more convenient. The only object of this paper was to show the existence of much simpler numerical solutions of the problems.
E. Volterra, USA

623. Re AMR 8, Rev. 3922 (December 1955): Setchkin, N. P., Self-ignition temperatures of combustible liquids.

The bibliographical heading for this review should read as follows: *J. Res. nat. Bur. Stands.* **53,** 1, 49–66, July 1954. The editors regret that the incorrect year of publication was given in the December issue. Ed.

Theoretical and Experimental Methods

(See also Revs. 641, 642, 643, 644, 650, 651, 653, 654, 663, 671, 688, 697, 699, 700, 779, 790, 798, 827, 852, 886, 887, 891, 919, 939, 959, 965)

624. Bacon, H. M., Differential and integral calculus, 2nd ed., New York, McGraw-Hill Book Co., Inc., 1955, vii + 547 pp. $6.

Author presents a standard treatment of the differential and integral calculus with the usual applications. Order of treatment is classical; integral calculus begins after differential calculus has been completed. Following the applications of simple integrals, there are chapters on partial differentiation, multiple integrals, infinite series, and differential equations. Second edition has been entirely reset with larger pages, and some details have been omitted. Thus 771 pages of the first edition have been reduced to 547 in the second; the new format is more pleasing than the original. There are many worked examples and large numbers of exercises. Author presents the traditional intuitive treatment rather than a mathematically formal one. G. B. Price, USA

625. Kaye, J., A table of the first eleven repeated integrals of the error function, *J. Math. Phys.* **34,** 3, 119–125, July 1955.

The tables contain values of the repeated integrals of the error function, $i^n \operatorname{erfc} x = {}_x\int^\infty i^{n-1} (\operatorname{erfc}\xi)d\xi$, where $\operatorname{erfc} x = 1 - \operatorname{erf} x$ and $\operatorname{erf} x = (2/\pi)\,{}_0\int^x e^{-\xi^2}d\xi$. Tabulated are $(2/\pi)e^{-x^2}$, erf x, erfc x, and i^n erfc x for $n = 1(1)$ and $x = 0.00(.01) \times 25(.05)1.00\ (.1)3.00$. The values are given to six decimal places with second differences for interpolation and extend to that value of x where the tabular entries are smaller than 10^{-6}. Applications of the tables are mentioned in the introduction.
M. Abramowitz, USA

626. Jacchia, L., On the numerical integration of functions tabulated in logarithmic form, *Math. Tables Aids Compt.* **9,** 50, 63–65, Apr. 1955.

Note considers application of finite difference formula for solution of title problem. Technique is useful when function values undergo rapid change. Author reports that method has been successfully employed to solve differential equations governing motion and loss of mass of meteors. Y. Luke, USA

✉627. **Besicovitch, A. S., Almost periodic functions,** New York, Dover Publications, Inc., 1954, xiii + 180 pp. $1.75 (paperbound).

If $f(x)$ is such that

$$f(x + \tau) = f(x)$$

to within arbitrarily small error for infinitely many values of τ, these values being spread over the whole range from $-\infty$ to $+\infty$ in such a way as not to leave empty intervals of arbitrarily great length, then f is almost periodic. In any particular application, an almost periodic function is experimentally indistinguishable from a strictly periodic one. The connection of the subject of this book with mechanics is thus obvious and is left to the reader.

The precise theory of almost periodic functions was created by Harold Bohr in 1924–1926 and is presented in his monograph, "Fastperiodische Funktionen" [Ergenbnisse der Math. 1, no. 5, Springer, Berlin, 1932; English translation by Harvey Cohn, Chelsea Publishing Co., N. Y., 1947]. The volume under review was first published by the Cambridge University Press in 1932. It is devoted to Bohr's theory but puts some emphasis on Besicovitch's treatment of the piecewise continuous case.

The reviewer regrets that the publisher of this reprint has given no indication of the date of the original. This omission is manifestly unfair to the author and confusing alike to librarians and to other writers citing the work. It would have added to the value of this reprint had a special preface, outlining progress subsequent to this old treatment and giving reasons for reissuing this classic, been added. Such a preface might have mentioned that, since Bohr's original papers, some 500 studies of the subject have been published. Von Neumann's theory of measure invariant over a group, created in 1933–1934 and making use of earlier ideas of Weyl, has permitted an abstract treatment which is both more general and shorter. For this theory, see chap. IV of Pontrjagin's "Topological groups" [Princeton, 1946], Example 46 being the fundamental theorem on almost periodic functions. The most recent treatise is that of W. Maak, "Fastperiodische Funktionen" [Springer, Berlin, 1950]. The old methods of Bohr, as presented in the present reprint, retain value for those interested in the concrete and calculational aspects of theory.

C. Truesdell, USA

628. **Schwarz, H. R., Differential equations. Stability criterion of systems with constant coefficients** (in French), *C.R. Acad. Sci. Paris* **241,** 1, 15–18, July 1955.

The system of equations considered is $y' = Ay$. The usual method of finding the roots of the characteristic equation of the matrix A is by-passed. Elementary transformations are applied which reduce A to a form to which a criterion due to H. S. Wall ["Continued fractions," D. Van Nostrand Co., p. 182, 1948] may be directly applied to determine the number of latent roots having a positive real part. Reviewer believes the method to be well worth a practical trial. R. P. Pearce, Scotland

629. **Mikeladze, Sh. E., Numerical solution of boundary problems for nonlinear ordinary differential equations** (in Russian), *Soobshchen. Akad. Nauk Gruzin SSR* **14,** 3, 133–137, 1953 (translated from Russian by M. D. Friedman, 572 California St., Newtonville, Mass., 6, pp)

Author describes a procedure for the reduction of a nonlinear ordinary differential equation to a family of nonlinear algebraic equations which can be solved by conventional iterative methods. The described procedure uses numerical quadrature rather than numerical differentiation. The method is used to compute the deflection of a rod, compressed by forces exceeding the Euler critical force.

Reviewer believes that the method may be of value in certain engineering applications and should complement other well-known numerical methods for the solution of boundary-value problems. It is likely that the convergence of the method may depend on the quadrature formulas used.

G. C. Wallick, USA

630. **Potters, M. L., A matrix method for the solution of a linear second-order difference equation in two variables,** *Math. Centrum Rep.* MR 19, 13 pp., 1955.

A method is presented for solving second-order partial difference equations in a rectangular region. Problem is written in form of second-order matrix difference equation with sets of values of unknown function at each successive station across the region as unknown column vectors. Ingenious numerical method for solving matrix difference equation directly by step-by-step procedure is explained. Method does not appear to be subject to the buildup of rounding-off errors that is often experienced with ordinary step-by-step approaches. Use of the method is not limited to problems considered by author; it should also be valuable for solving coupled sets of second-order difference equations in many unknowns. J. M. Hedgepeth, USA

631. **Mayr, S., Complex vectors in matrix representation** (in German), *Electrotech. u. Maschinenb.* **72,** 19, 460–464, Oct. 1955.

✉632. **Webster, A. G., Partial differential equations of mathematical physics (ed. by Plimpton, S. J.), 2nd cor. ed.,** New York, Dover Publications, Inc., 1955, vii + 440 pp. $1.98 (paperbound).

✉633. **Miranda, C., Partial differential equations of elliptical type [Equazioni alle derivate parziali di tipo ellittico],** Berlin, Springer-Verlag, 1955, 222 pp. DM 28.80.

Book gives in condensed form an excellent survey of the general theory with special emphasis on the existence theorems and the nature of solutions under given boundary conditions. In the first two chapters, the necessary definitions and mathematical tools are presented and, in the third, two methods of solution by transforming the classical boundary-value problems into integral equations of the second kind are discussed. The next chapter is devoted entirely to the behavior of solutions if more general conditions are prescribed for coefficients of the equation, for the domain, or for the boundary values. Some methods based on various procedures of the linear functional analysis are explained, and then the methods of transforming boundary-value problems into integral equations of the first kind are introduced, together with some hints on the possibilities of using them for actual determination of solutions. The next chapter is devoted to general methods of determining as closely as possible the bounds for solutions of the problem of Dirichlet, and the sixth gives the extension of the theory for nonlinear equations in the case of boundary-value problems of Dirichlet and Neumann. The last chapter gives some hints and bibliographic data on further extensions of the theory. Among them some generalized boundary-value problems are briefly mentioned and systems of equations and certain equations of higher order are also treated, together with hints on their importance in the theory of elasticity.

Book omits, of course, all purely numerical methods and gives only bibliographical data for the variational method and for the method of the kernel function, as there exist modern treatises on both. Different methods for the solution of classical boundary-value problems are compared, and, in some instances, many details are cleared up. Book is intended primarily for the theoretician, but reviewer finds some problems of interest for the research worker in applied mechanics too, in so far as he can follow the brief, though very clear, treatment of a difficult mathematical subject. A. Kuhelj, Yugoslavia

634. Ho, K.-C., Double interpolation formulae and partial derivatives in terms of finite differences, *Math. Tables Aids Compt.* **9,** 50, 52–62, Apr. 1955.

Paper tabulates a wide variety of formulas identified in title. These are useful for interpolation and numerical solution of partial differential equations. Y. Luke, USA

635. Harkeevič, Y. F., Graphical methods of solution of integral equations (in Russian), *Inzhener. Sbornik, Akad. Nauk SSSR* **15,** 207–215, 1953.

In this paper, author gives certain geometrical interpretations of Fredholm and Volterra integral equations from which he develops a method for the graphical solution of these equations. His method carries over to a graphical procedure the familiar analytic method of successive approximations. He employs the procedure of A. B. Štykan [title source, **13,** 177–186, 1952] for the construction of the integral of the product of two functions.
Courtesy of Mathematical Reviews W. E. Milne, USA

636. Kraus, R., Angle and locus curves in computing linkages (in German), *Feinwerktech.* **58,** 6, 186–189, June 1954.

A function can be approximated by a four-link mechanism that can be calculated by knowing the function and its first four derivatives. The method presented here reduces this by one, thus simplifying the calculations. Freedom is also allowed for adjusting the various links so as to secure a mechanism which will be practical to operate as well as theoretically exact. The method involves determining two curves or loci, which then permit construction of other linkages which will each generate the desired function. C. E. Balleisen, USA

637. Hain, K., Development of computing linkages (in German), *Feinwerktech.* **58,** 6, 190–198, June 1954.

An interesting survey of the development of linkages for computing purposes over the past two decades. Excellent and valuable bibliography of this subject (approximately 140 entries) is appended. The construction of a six-link mechanism having uniform input and output scales is described.
C. E. Balleisen, USA

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 636, 637, 647, 671, 686, 696, 697, 728, 729, 837, 839, 851, 951, 953)

638. Byrum, B. L., and Grady, E. R., General air-frame dynamics of guided missile, *J. aero. Sci.* **22,** 8, 534–540, Aug. 1955.

Equations of motion of rigid airplane-type vehicle of constant mass and in constant gravity field are formulated in two ways. First describes motion of principal body axes of inertia, and second describes motion of relative wind axes, both with reference to inertial system of axes. It is concluded that in the general three-dimensional case, body axis formulation appears to be more tractable to solution by computing machines.
L. Becker, USA

639. Huszthy, L., Computation of the inertia moment of an evolute tooth (in Hungarian), *Gép* **7,** 8, 305–308, Aug. 1955.

Author computes "exact" value of moment of inertia for gears with evolute teeth. He shows that the error is 53% if the equivalent solid ring is taken for moment of inertia calculations, and 7.3% if half of the ring (from root to outer radius of teeth) is taken. However, the total error seems to be less than given by author, since the contribution of the teeth-ring is only a part of the total moment of inertia of the gear. T. P. Torda, USA

640. Thiry, R., Work of internal forces in a continuous medium during deformation (in French), Fluid Mech. Meeting (Marseille, 1952), *Publ. sci. tech. Min. Air. France* no. 296, 233–242, 1955.

Remarks with respect to a paper by H. Beghin ["Sur la notion de travail dans la mécanique du continu," *Ann. Inst. Fourier Univ. Grenoble* **II,** 173–184, 1950] on the computation of the energy of deformation of an elastic body. A discussion on the subject is added in which Messrs. Pérès, Martindt-Lagarde, and Vernotte take part. C. B. Biezeno, Holland

Servomechanisms, Governors, Gyroscopics

(See also Rev. 843)

641. Truxal, J. G., Automatic feedback control system synthesis, New York, McGraw-Hill Book Co., Inc., 1955, xiii + 675 pp. $12.50.

This is an advanced text on automatic control theory which contains a good selection of well-organized topics. Author includes many recent advances of the art, some orginated by himself, which are brought together here for the first time in a book. This material is integrated into a somewhat unconventional but complete picture of automatic control. Of special interest are the extensive use of synthesis methods based on root locations, including Guillemin's synthesis based on the preselection of closed loop roots; the unusual extent to which network synthesis techniques are utilized beyond the application of simple lead and lag networks; the use of Mason's signal flow graphs; the use of two-sided Laplace transforms and Z transforms, etc.

The book contains chapters on signal-flow diagrams and feedback theory, synthesis of RC networks, root-locus methods, synthesis through pole-zero configurations, design in the S-plane, principles of statistical design, application of statistical design principles, sampled data control systems, nonlinear systems and describing function analysis, phase-plane analysis.
J. Zaborszky, USA

642. Draper, C. S., McKay, W., and Lees, S., Instrument engineering, Vol. 3, Applications of the instrument engineering method; Part I, Measurement systems, New York, McGraw-Hill Book Co., Inc., 1955, xxviii + 879 pp. $17.50.

The previous two volumes by the same authors have been concerned with fundamental theory, whereas the present one is devoted to the problem of measurement. The case method is employed with emphasis on temperature, pressure, and vibration. Physical components are described by drawings of equipment, differential equations, and responses to sinusoidal, ramp, step, and pulse inputs. The style follows that of the previous volumes. Reviewer believes the book will be a valuable source of information for both the beginner and expert.
R. Oldenburger, USA

643. Boonton, R. C., Jr., The analysis of nonlinear control systems with random inputs, *Proc. Symposium on nonlinear circuit analysis,* 1953; Polytechnic Inst., Brooklyn, 1953, 369–391.

The behavior of nonlinear elements of control systems is studied in a statistical way. An ensemble of input functions $x(t)$ of time t is characterized by probability density functions; the problem is to obtain useful information about the statistical properties of the ensemble of output functions $y(t)$. As a typical example, a limiter element where $y(t) = f(x(t))$ and $f(x) = c_1 \min(|x|, c_2) \operatorname{sgn} x$ is considered; c_1 and c_2 are positive constants. The output is rewritten as

$$y(t) = Kx(t) + x^*(t)$$

with a constant K (the equivalent gain); K is determined as the minimizing parameter of the integral

$$M = \int_{-\infty}^{\infty} (f(x) - Kx)^2 p_1(x) dx$$

with p_1 as the first probability density function of the ensemble $x(t)$. For practical predictions on the behavior of the nonlinear element, the "distortion" x^* is neglected. This means that the element is replaced by an "equivalent" linear element with $y = Kx$ as relation between input and output. The paper goes into numerical details on K for Gaussian ensembles $x(t)$ and also presents results on feedback control systems with a limiter element. Author indicates how the basic idea of the equivalent linear element can be generalized; he also points out that the investigations should be extended to other typical nonlinear elements, especially to those with backlash. Reviewer believes that a promising start has been made. H. Bückner, USA

644. **Bottema, O., On the stability of the equilibrium of a linear mechanical system,** *ZAMP* **6,** 97–104, 1955.

The differential equations of a linear mechanical system of two degrees of freedom are investigated for stable equilibrium. Author is particularly interested in the case in which unstable equilibrium approaches stability as the "damping" forces tend to zero.

Courtesy of Mathematical Reviews E. Pinney, USA

645. **Csordás, Z., Pneumatic control** (in Hungarian), *Mérés és Automatika* **8,** 9, 232–241, Sept. 1955.

This is a well-written and up-to-date survey of pneumatic control instruments based entirely on recent U.S., German, British, and Russian publications, covering theoretical, technological, and practical aspects. Such instruments are not now manufactured in Hungary and no references to Hungarian literature are given.

R. E. Kalman, USA

646. **Stürmer, W., Proportional control of water levels by means of servomotors with constant speed** (in German), *Brennstoff-Wärme-Kraft* **7,** 7, 309–312, July 1955.

A control system with the differential equations $s(dx/dt) = z - y$; $dy/dt + ay = bx$; (a, b, s are positive constants, x water level, y valve position, z perturbation, t time) is considered under the assumption that a limitation $|dy/dt| \leq M$, due to the finite maximum speed of a servomotor, occurs. The behavior of the control system is studied for the case z = const, especially after the limit for $|dy/dt|$ is reached. There are some numerical examples. H. F. Bückner, USA

647. **McCarthy, W. E., and Van Wicklin, W. A., Hydraulic power steering-effects of valve characteristics and steering wheel inertia,** ASME Advance paper no. 55—A-64, 4 pp. + 6 figs., 1955.

Paper discusses the dynamics of a hydraulic steering system for passenger automobiles. The automatic feedback-control system is outlined in detailed block-diagram form. Each element of the system is briefly described without mathematical derivations. Two blocks of the system are more closely examined. The steering-wheel inertia has an important effect on the reversibility of the system. This is shown in the block diagram and by experimental test data. System damping resulting from the hydraulic control-valve characteristics and the relations between steering wheel velocity and output are described.

From authors' summary

648. **Bock, H., Tolerance of controlled quantities** (in German), *Regelungstech.* **3,** 11, 272–275, 1955.

It is shown that the customary treatment on controlled quantities by means of the frequency spectrum generally does not yield any data concerning the bounded peak deviation. The reason for this is explained. Since this question is of great practical interest and the limitation often really does exist, the treatment by statistical error margins is briefly explained, and the measuring by means of integrating measuring equipment is suggested. The necessity of treating spatial tolerances differently is referred to.

From author's summary

649. **Fabritz, G., Control of gas turbines** (in German), *Regelungstech.* **3,** 11, 266–268, 1955.

The definition in principle of gas turbine controls depends upon the system applied. The paper is a discussion, with the aid of diagrams, of controls of systems which work on the equal pressure principle and are of the two-shaft design. Controls of the open circuit, as well as the closed circuit of the Ackert-Keller design, are dealt with. From author's summary

Vibrations, Balancing

(See also Revs. 643, 731, 780, 786, 854, 929, 930, 938, 956, 957)

650. **Mazet, R., Mechanical vibrations [Mécanique vibratoire], Liège and Paris, Librairie Polytechnique Ch. Béranger, 1955,** xix + 280 pp. 4975 Fr.

This is a text which treats vibration analysis in an orderly fashion from undamped one-degree-of-freedom systems on up to self-excited and nonlinear systems. The methods used are principally those of electric circuit theory translated into mechanical terminology. The reader is supposed to be familiar with Lagrange's equations for a dynamic system. The operational method (Laplace transforms) for step and impulse responses and the impedance method for steady-state sinusoidal responses are developed in the text. Matrix notation is used for systems with many degrees of freedom.

Author gives the name "global method" to the process of selecting a hypothetical mathematical model for a real physical system and then determining the coefficients in the mathematical model by measuring the response of the real system to certain specialized excitations. This idea is used as a unifying device throughout the book, although the book itself treats only the vastly simpler problem of calculating the response of given mathematical models. There is a chapter on coupling, including elastic coupling, inertia coupling, gyroscopic coupling, and electromechanical coupling. There is a chapter on ideal filter theory applied to mechanical systems.

Reviewer considers the treatment a little too sketchy to make it a useful text. Many complicated developments are only vaguely suggested and references are not usually given to more complete treatments. Most developments remain in a generalized form, stopping short of particular applications. There is only one numerical illustration in the book and that contains an example of poor computational practice. (Having obtained the matrix of influence coefficients at a number of stations along a uniform beam he *inverts* this and *then* finds the eigenvalues in order to get the natural frequencies. The unnecessary inversion makes accurate determination of the important low frequencies more difficult.)

S. H. Crandall, USA

651. **Minorsky, N., On nonlinear resonance** (in French), *C. R. Acad. Sci. Paris* **240,** 26, 2482–2484, June 1955.

The stroboscopic method is applied in order to explain the discontinuities in the curve of nonlinear resonance. Author shows that these discontinuities are expressible by the roots of a cubic equation. G. Sestini, Italy

652. Richards, D. A., Forced vibration with damping, *Amer. J. Phys.* **23,** 9, 557–561, Dec. 1955.

The experimental determination of the response curves for forced vibration with velocity damping, using a moving-coil galvanometer, is described. The mechanism for producing a sinusoidal emf, of continuously variable frequency ranging from zero to 0.4 cps, is described, together with a method for the determination of the phase angle between the driving force and the motion of the driven system. Experimental results, obtained with the apparatus, both for response curves and phase angle measurements, for various values of damping, are summarized.

From author's summary

653. Gillies, A. W., The periodic solutions of a non-linear differential equation of the second order with unsymmetrical non-linear damping, and a forcing term, *Quart. J. Mech. appl. Math.* **8,** part I, 107–128, Mar. 1955.

A study of the equation

$$\ddot{v} - \omega_0(\epsilon + \epsilon^{1/2}kv - \epsilon v^2)\dot{v} + \omega_0^2 v = 2B\,\omega_1 \sin \omega_1 t$$

where ϵ is small and B and $\omega_1 - \omega_0$ are $O(\epsilon)$, is made by series development. Convergence for ϵ sufficiently small is established by comparison with a convergent series of positive constant.

Resonance curves for periodic solutions correspond more closely to those obtained experimentally than van der Pol resonance curves. H. G. Lew, USA

654. Popov, E. P., On the approximate study of self and forced oscillations on nonlinear systems (in Russian), *Dokladi Akad. Nauk SSSR* **95,** 5, 943–946, Apr. 1954 (translated from Russian by M. D. Friedman, 572 California St., Newtonville, Mass. 6 pp.).

See AMR **8,** Rev. 1578.

655. Kordes, E. E., and Kruszewski, E. T., Investigation of the vibrations of a hollow thin-walled rectangular beam, *NACA TN* 3463, 24 pp., Oct. 1955.

A rectangular hollow box beam of uniformly thin material without internal stiffening ribs has been tested to find its first five modes and frequencies in each of bending and torsion. Bending frequencies have been compared with those given by elementary theory and that of AMR **6,** Rev. 1173. Predictions are not accurate within 10% above second mode. Method of AMR **7,** Rev. 3482, however, gives agreement within 5% for all five modes. For torsion, elementary theory and that of AMR **8,** Rev. 2982 give errors greater than 20% for all frequencies. Method of *NACA TN* 3464 (see following review) gives accuracy within 9% for all five modes when experimental values of cross-sectional shear stiffnesses are used. Results underline need for allowing for cross-sectional deformation in calculating frequencies of hollow beams. K. H. Griffin, England

656. Kruszewski, E. T., and Davenport, W. W., Influence of shear deformation of the cross section on torsional frequencies of box beams, *NACA TN* 3464, 23 pp., Oct. 1955.

Effect of shear stiffness of bulkheads on the torsional vibrations of four-flange box beams is investigated using energy methods. Analysis permits general cross-sectional distortions of a shearing nature. Solutions are presented for a uniform free-free beam, and curves are included for a specific geometry. In addition, an approximate method is given for quickly estimating the effect of bulkheads on frequencies. Author concludes that the influence of cross-sectional shear upon torsional frequency is generally negligible for box beams of conventional construction having bulkheads spaced approximately one chord apart; however, the influence could be important for wing designs. A. I. Bellin, USA

657. Newman, N. K., Effect of rotatory inertia and shear on maximum strain in cantilever impact excitation, *J. aero. Sci.* **22,** 5, 313–320, 348, May 1955.

Problem of impact of a cantilever beam subjected to a half-sine wave excitation applied at the root is studied analytically. Laplace transformation method is used to solve the Timoshenko equation; special attention is given to the formulation of the appropriate boundary conditions. Results are given in the form of a strain amplification factor and are discussed adequately. Higher transmission mode of the Timoshenko theory is not considered. H. N. Abramson, USA

658. Busemann, K., Shifted logarithmic spirals for the plotting of damped vibrations and their derivations for the calculations of beams on elastic foundation and for the calculation of circular cylindrical shells with uniformly distributed symmetrical loads (in German), *Stahlbau* **24,** 4, 73–77, Apr. 1955.

A graphical method using logarithmic spirals $r = r_0 e^{-u}$ has been developed for the calculation of the internal forces of a beam on elastic foundation. Various cases of the loading of the beam are discussed. Author considers the wall of a circular cylindrical shell to be composed of straight beams resting on circular beams. By regarding the circular beams as an elastic foundation for the straight ones, author applies his graphical method to calculate the internal forces in a uniform symmetrically loaded cylindrical shell. J. W. Cohen, Holland

659. Abramowitz, M., and Cahill, W. F., On the vibration of a square clamped plate, *J. Assn. Comp. Machy.* **2,** 3, 162–168, July 1955.

The method of finite differences with better approximations to the boundary conditions is utilized. Results indicate more significant improvements in accuracy by this means than by the decrease in mesh width; for a fixed mesh width there is an optimum rule to be used. H. G. Lew, USA

660. Martinek, J., and Yeh, G. C. K., Sound scattering and transmission by thin elastic rectangular plates, *Quart. J. Mech. appl. Math.* **8,** part 2, 179–190, June 1955.

The work of Vogel on an ideal elastic plate simply supported in an infinite baffle is extended in two ways: (1) the incident wave (although separable into Cartesian coordinates) need not be plane; (2) the fluid media on opposite sides of the plate may be different. The incident wave excites forced vibrations of the plate, and the resulting scattered wave is expressed in terms of the constants of the plate and the media. The plate (composed of material which is isotropic, devoid of damping, and which obeys Hooke's law) is treated essentially as a plane in relating the normal velocity and excess pressure at its surface to those of the assumed sound field; the media it separates are specified by the small amplitude wave equations. The excess pressure of the field is then taken as the inhomogeneous term of the fourth-order equation for the transverse deflections of the plate, and the corresponding Lagrange equations are used to determine the motion. Expressions are given for the normalized time-averaged transmitted energy, the energy absorbed by the plate, and for the reaction forces at the plate supports. V. Twersky, USA

661. Cox, H. L., and Klein, B., Fundamental frequencies of clamped triangular plates, *J. acoust. Soc. Amer.* **27,** 2, 266–268, Mar. 1955.

Paper determines fundamental mode and frequency of a triangular plate with clamped edges by means of the approximate method of collocation. A displacement mode which satisfies the boundary condition and which depends linearly upon two

arbitrary parameters is made to satisfy the plate differential equation at two points on the bisector of one apex of the triangle. Resulting frequency equation is solved and results are presented graphically. W. S. Hemp, England

662. **Thurston, E. G., and Tsui, Y. T., On the lowest flexural resonant frequency of circular disk of linearly varying thickness driven at its center,** *J. acoust. Soc. Amer.* **27,** 5, 926–929, Sept. 1955.

Rayleigh-Ritz method is used, assuming displacement may be expressed as third-degree polynomial in r, the radius. Author states that center of plate, in the practical case that he dealt with, was neither fixed nor free. Reviewer assumes it had spring constraint. Refinement of method seems pointless until stiffness of spring constraint is specified. D. C. Johnson, England

663. **Jasper, N. H., A theoretical approach to the problem of critical whirling speeds of shaft-disk systems,** *David W. Taylor Mod. Basin Rep.* no. 827, 31 pp., Dec. 1954.

A number of practical methods are presented for numerical calculation of natural frequencies of shaft-disk systems to include gyroscopic effects. Included are explicit methods for single disk, trial frequency method, and electrical analog. Emphasis is on formulation for digital calculators. Note erratum sheet. R. Plunkett, USA

664. **Niordson, F., Vibrations of turbine blades with loose hinge support,** *Acta Polyt.* (*Mechanical Engineering Series* **3, 3**), pt. 140, 11 pp., 1954.

Equations of motion are established for blade in centrifugal field. Solution for fundamental mode is obtained by modified Vianello-Stodola method. Oscillation becomes superposition of pendulum and bending oscillation. Example including several limiting subcases is presented, giving fundamental frequency vs. rotational speed. J. H. Greidanus, Holland

665. **Cohen, E., Levy, L. S., and Smollen, L. E., Impulsive motion of shear buildings including plasticity and viscous damping,** *Proc. Amer. Soc. civ. Engrs.* **81,** Separ. no. 676, 1–19, Apr. 1955.

The equations of motion for a multistory shear building with impulsive loads are developed in terms of normal modes of vibration. The development of plastic hinges between stories in building on rigid foundation and temporary instability and plasticity in the foundation soil is discussed. Rotation of building with large plastic distortions in superstructure is described and a general approach to solution is outlined. Approximate modes and frequencies of some typical shear buildings having two to five stories are tabulated. P. G. Jones, USA

666. **Suetsugu, I., A contribution to the vibration at the stern of a single screw vessel,** *Inter. Shipbldg. Progr.* **2,** 10, 275–282, 1955.

Author considers vibration of engine and hull due to horizontal components of thrust from a single screw. He solves equations appropriate to an elastic beam (hull) loaded through a spring (mounting) by a mass (engine) and subjected to lateral thrusts originating from the blades of the screw; he shows normal modes to exist in which amplitude of vibration is significant only at the stern—like a fish wagging its tail. The frequency of excitation is (*rpm* × *the number of blades on the screw*); when this product is near to the appropriate natural frequency of the hull, large amplitudes of stern of hull and of engine are observed.

Remedy may be effected by altering the dynamic characteristics of the system or, more satisfactorily, by reducing the lateral thrust from screw by regulating flow conditions.

Second alternative has been successfully adopted for ore carriers on the Great Lakes, according to the only reference given [*Trans. Soc. nav. Arch. mar. Engr.* **60,** 1952]. M. J. P. Musgrave, England

667. **Arthur, B., Computation of natural frequencies of torsional vibrations with aid of characteristic equations** (in Hungarian), *Magyar Tud. Akad. Oszt. Közl.* **15,** 1/2/3/4, 91–113, 1955.

In calculations of torsional vibrations, one has to start from the determination of the natural frequencies and the elements have to be dimensioned accordingly. If, however, the characteristic equation is to be used for the determination of the natural frequency, then, already for the case of eight masses, the calculations are very complex. In order to avoid the difficult computations, solutions have been found which avoid the use of the characteristic equation.

In the present paper, a method is given which makes use of the characteristic equation possible for the determination of the natural frequency in a simple manner for an arbitrary number of masses. From author's summary by T. P. Torda, USA

668. **Robey, D. H., Theory of the effect of a thin air film on the vibrations of a stretched circular membrane,** *J. acoust. Soc. Amer.* **26,** 5, 740–745, Sept. 1954.

The dynamics of a condenser microphone with plane rigid fixed electrode has been worked out. The discussion, based on linearized Navier-Stokes equation, leads to evaluation of accession to inertia and resistance of air trapped between electrodes. Author computes air reactions for case of a stretched titanium membrane and finds that amplitude response of microphone drops by 20 db in range 5 to 20,000 cps because of continuously increasing effective stiffness of air film. O. K. Mawardi, USA

669. **Lewis, F. M., The extended theory of the viscous vibration damper,** *J. appl. Mech.* **22,** 3, 377–382, Sept. 1955.

Paper applies to the design of a tuned or untuned vibration damper for a multimass system. Author shows that, in such a system, the displacement of any mass is independent of the damping constant of the absorber at certain frequencies. From this, special formulas are derived for the design of nearly optimum dampers for the two mass systems. A. H. Hausrath, USA

Wave Motion in Solids, Impact

(See also Rev. 657)

670. **Miklowitz, J., Traveling compressional waves in an elastic rod according to the more exact one-dimensional theory,** *Proc. second U.S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 179–186.

Solution is obtained by use of Laplace transforms to problem of suddenly applied constant axial force at end of semi-infinite circular rod, according to Mindlin-Hermann approximate theory. The end of the rod is supposed constrained against radial expansion.

The traveling wave is the sum of two waves whose leading edges travel at speeds equal to the limiting speeds for very short wave lengths of the two modes of motion considered by Mindlin and Hermann. A related finite rod solution is also obtained. Solutions contain certain improper integrals whose numerical evaluation is required for comparison with experimental data; such evaluations are reported to be in progress.

Reviewer notes one misprint, omission of factor η in left member of last equation in first column, p. 180. L. Malvern, USA

Elasticity Theory

(See also Revs. 640, 685, 693, 698, 701, 706, 724, 728, 732, 734, 736, 740)

671. Thomas, T. Y., On the structure of the stress-strain relations, *Proc. nat. Acad. Sci.* **41,** 10, 716–720, Oct. 1955.

It is well known that, in the Eulerian description of the motion of a continuum, the acceleration or material rate of velocity is the sum of the local and convective rates. It is not generally understood, however, that further terms must be added in the definition of the material rate of a tensor of the second or a higher order. The problem treated in the present paper provides an illustration. Let the mechanical behavior of a material be defined by giving the "stress rate" as a function of the stress and the velocity strain. If this stress-strain relation is to make sense physically, the "stress rate" must be interpreted as the material rate of stress. This is the sum of the local, convective, and "rotational" rates of stress (in the terminology that this reviewer has for several years used in his lectures). The need for the convective and rotational terms becomes evident when the stress-strain relation is applied to a rigid-body translation or rotation of a stressed element. A mathematical expression for the rotational rate of stress is derived which, except for notation, agrees with Fromm's [*Ing.-Arch.* **4,** p. 452, 1933]. It is pointed out that alternative expressions, e.g., that given by Truesdell [*Comm. pure appl. Math.* **8,** p. 123, 1955], differ from the expression derived here only by terms which involve stress and velocity strain and could, therefore, enter the stress-strain relation in their own right.

W. Prager, USA

672. Thomas, T. Y., Combined elastic and Prandtl-Reuss stress-strain relation, *Proc. nat. Acad. Sci.* **41,** 10, 720–726, Oct. 1955.

Author proposes to establish a single set of stress-strain relations that is valid in both the elastic and the plastic range. Starting from the assumptions that the stress rate is a function of the stress and the velocity strain, and the stress intensity is limited by the Mises yield condition, he introduces a number of simplifying assumptions which lead to a comparatively simple stress-strain relation for a material that exhibits a gradual transition from the elastic to the plastic state. While the paper contains no statement to this effect, this stress-strain relation is presumably meant only for "loading" (positive rate of work) and should be supplemented by the usual differential form of Hooke's law for unloading. (For an incompressible material, author's stress-strain relation agrees with that used for loading in several papers of this reviewer; see *Duke Math. J.* **9,** p. 228, 1942, where further references are given). W. Prager, USA

673. Marguerre, K., Methods of solution of the basic equations in the theory of elasticity (in German), *ZAMM* **35,** 6/7, 242–263, June/July 1955.

Author presents a comprehensive and critical review and study of methods of attack used in about fifty papers for solving two- and three-dimensional elasticity problems, including special cases of axial symmetry and torsion, in the absence of body forces and temperature effects. Relations among various methods are emphasized. The very complete coverage of important results in the literature makes paper most valuable for future workers in the field.

Author begins discussion with two-dimensional methods, preparing for extension into three dimensions. The important work of Muskhelishvili, which is not so much related to works of others, is thus only very briefly mentioned. The displacement or strain-function approach used in three-dimensional cases is next discussed around works of Boussinesq, Love, Galerkin, Papkovitch, Neuber, Mindlin, Sadowsky and Sternberg, and others. Discussion on the stress-function approach in three-dimensional problems is the most interesting part of paper; in it, author presents a new method using the Maxwell and Morera stress functions. In vector and tensor notations and through special arrangement, the three-dimensional compatibility equation in terms of these stress functions is shown to appear in a form similar to the two-dimensional one in terms of the Airy stress function; namely, the second-order symmetric tensor having these stress functions as components satisfies the biharmonic equation. Since this tensor actually degenerates into the Airy stress function, the two-dimensional case is virtually extended into the three-dimensional. (The recent work by Langhaar and Stippes [AMR **8,** Rev. 1940] on the Maxwell and Morera functions could be added to author's bibliography.)

Final part of paper is on torsion. Vector and tensor notations used in paper are explained in an appendix.

Y.-Y. Yu, USA

674. Reiner, M., The complete elasticity law for some metals according to Poynting's observations, *Appl. sci. Res.* (*A*) **5,** 4, 281–295, 1955.

Stress-strain relation of Reiner [*Amer. J. Math.* **70,** 433, 1948; AMR **2,** Rev. 574] is solved in closed form for torsion of circular cylinder. It is examined in relation to Poynting's experiments, which it confirms. Numerical values of modulus of dilatancy are calculated for steel and hard copper.

W. Freiberger, USA

675. Radok, J. R. M., Problems of plane elasticity by reinforced boundaries, *J. appl. Mech.* **22,** 249–254, 1955.

Contrary to the method of the indirect approach of certain authors [e.g., Timoshenko, Gurney, Beskin, Wells, Reissner], in this paper, based on Muskhelishvili's monograph on plane elasticity ["Some basic problems of the mathematical theory of elasticity," 3rd ed., Moscow-Leningrad, Izdat. Akad. Nauk SSSR, 1949; AMR **4,** Rev. 577], a general direct method for problems of reinforced cutouts in infinitely thin sheets is deduced. This case is frequent in aircraft construction. The method has the advantage of being applicable to all types of holes which may be, exactly or approximately, mapped conformally onto the unit circle. As an example the problem of the reinforced circular hole is treated in detail numerically. Stress and strain concentration factors are presented and shown graphically. The results of author's method agree better with experimental results than with those of Reissner and Morduchow [*NACA TN* **1852,** 1949]. Simultaneously an identical method has been used by Savin ["Concentration of stresses around openings," Moscow-Leningrad, Gosud. Izdat. tekh.-teor. Lit., 1951; AMR **5,** Rev. 1318].

D. Rašković, Yugoslavia

676. Mossakovskii, V. I., Pressure on an elastic half-space of a die which is almost circular in planform (in Russian), *Prikl. Mat. Mekh.* **18,** 675–680, 1954.

The area of contact of the die with the half-space is assumed to have a center of symmetry and the contact is assumed to be smooth. The problem in which the area of contact is replaced by its largest inscribed circle is solved by the use of certain contour integral representations given by the author in another paper [title source, **18,** 187–196, 1954; see AMR **8,** Rev. 1597], and this solution is used to obtain an approximate solution of the given problem. The method is applied to an elliptic area of contact.

L. M. Milne-Thomson, England

677. **Rostovcev, N. A., On the problem of torsion of an elastic half-space** (in Russian), *Prikl. Mat. Mekh.* **19,** 55–60, 1955.

Paper contains a solution of an axially symmetric problem concerned with the determination of stresses and displacements in an elastic half-space twisted about the axis of symmetry by a rigid stamp connected with the half-space by friction or cohesion.

I. S. Sokolnikoff, USA

678. **Raman, C. V., and Viswanathan, K. S., The elastic behaviour of isotropic solids,** *Proc. Indian Acad. Sci.* (*A*) **42,** 1, 1–9, July 1955.

Authors state that "the stresses in an elastic solid cannot be assumed to be exclusively in the nature of *tractive forces* but may also include *torques*." But they do not define how these torques are related to the elements.

From conditions of symmetry and especially from considerations of symmetry under rotations about three perpendicular axes, authors deduce the expression for the deformation energy involving three constants. Their conclusion is that "the elastic behavior of the isotropic solid requires three independent constants for its description." It is, however, seen that infinitesimal theory which authors have adopted leads to only two strain invariants of the second degree. Necessarily only two independent constants are expected in the energy function for an isotropic solid. How this fact can be reconciled with the theory of the authors is not quite clear. S. C. Das, India

679. **Pfluger, A., The orthotropic plate with reinforcements in one direction** (in German), *Öst. Ing.-Arch.* **9,** 2/3, 199–207, 1955.

Author considers a rectangular plate of uniform thickness and bearing in one direction a series of reinforcements each of which, together with the plate, forms a closed torsionless resistant hollow cross section. The differential equations for the displacements are derived for the orthotropic plate and the solutions are discussed. A confirmation of the theory is obtained from measurements conducted on a model. J. J. Brandstatter, USA

680. **Adkins, J. E., A note on the finite plane-strain equations for isotropic incompressible materials,** *Proc. Camb. phil. Soc.* **51,** part 2, 363–367, Apr. 1955.

681. **Hilton, H. H., Thermal stresses in thick-walled cylinders exhibiting temperature-dependent viscoelastic properties of the Kelvin type,** *Proc. second U. S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 547–553.

The problem solved is that of steady-state temperature stresses in the incompressible linear Kelvin body. Since the Kelvin body represents at best the behavior of a real metal under conditions of adiabatic oscillations, the formulated and solved problem is mainly of theoretical interest. A. M. Freudenthal, USA

682. **Salvadori, M. G., Live load and temperature moments in shells of rotation built into cylinders,** *J. Amer. Concr. Inst.* **27,** 2, 149–158, Oct. 1955.

Maximum bending moments are calculated for variety of shells of revolution under axial loading and temperature variation. For shells built in to cylinders of same radii, maximum moments are functions of shell-cylinder thickness ratio. Factors for evaluation are given in tabular form. R. N. Arnold, Scotland

683. **Lomakin, V. A., Elastic-plastic equilibrium of a sphere in an unsteady temperature field** (in Russian), *Prikl. Mat. Mekh.* **19,** 244–248, 1955.

A stress-free spherical mass is at a uniform temperature. The surface temperature is suddenly decreased and is kept constant. In this paper, attention is directed toward the theoretical analysis of the subsequent elastic-plastic deformation of the mass. Stress equilibrium is assumed, and a finite-strain theory of plasticity is used. The physical constants specifying mechanical behavior are supposed independent of temperature. Numerical results are presented in graphical form.

H. G. Hopkins, England

684. **Landauer, J. K., Stress-strain relations in snow under uniaxial compression,** *J. appl. Phys.* **26,** 12, 1493–1496, Dec. 1955.

Two types of tests were performed on the uniaxial compression of snow; experiments at constant strain rate and at constant load. The strain rate can be represented as the product of functions of density, stress, temperature, and time. For densities between $\gamma \simeq 0.3$ g/cc and $\gamma \simeq 0.6$ g/cc, the strain rate is proportional to $(\gamma_{ice} - \gamma)/\gamma$ to the fourth power. The strain rate varies linearly with stress for low stresses ($2 < \sigma < 100$ g/cm^2) but increase to an approximately cubic relationship at higher stresses ($1 < \sigma < 5$ kg/cm^2). An activation energy of about 14,000 cal/mole can be derived from the temperature dependence. In the constant load tests, the strain is proportional to the time to a power of about 0.8. The effect of snow type is small, being less than the relatively large experimental scatter encountered.

From author's summary

Experimental Stress Analysis

(See also Rev. 732)

685. **Perry, C. C., and Lissner, H. R., Stress equations for strain gage rosettes,** *Prod. Engng.*, **27,** 7, 207–211, July 1955.

Paper summarizes stress equations for four types of electric strain-gage rosettes (two-gage, rectangular, delta, and T-delta).

Reviewer considers it a convenient reference for strain-gage users. A.-T. Yu, USA

686. **Buckner, H. A., and Rebeske, J. J., Application of high-speed strain-gage torquemeter to turbomachinery research,** *Trans. ASME* **77,** 5, 597–603, July 1955.

This paper presents a description of a high-speed electrical strain-gage torquemeter. High accuracy required for the research is claimed. Shows application of the torquemeter to compressor and turbine setups. Also discusses pertinent factors involved in obtaining accurate measurements with the instrument described. C. O. Dohrenwend, USA

687. **Martinaglia, L., Strain gauge measurements on high-lift storage pumps,** *Sulzer tech. Rev.* **37,** 1, 32–40, 1955.

Since the introduction of the wire resistance strain gage for measuring stresses in engineering components, numerous tests on pump casings and covers have been carried out with this device and thereby information of great value in the calculation and designing of components for high-lift pumps has been acquired.

After a short description of the method, the article gives an account of measurements made on a storage pump.

From author's summary

688. **Favre, H., Application of the least-squares method for the solution of the system of equations used in photoelasticity** (in French), *Rev. Opt.* (*théor. Instrum.*) **34,** 6, 305–322, June 1955.

In using the author's well-known method in problems of plane stress, the relative retardation and two absolute retardations are measured. Each of these three quantities is a linear func-

tion of the two principal stresses. Accordingly, a redundant (and slightly inconsistent) set of equations is obtained for the stresses. Author shows that least-squares analysis can considerably reduce the mean errors in the calculated principal stresses below the mean errors resulting from arbitrary combinations of the equations. Working formulas are developed for application to plane stress problems and the corresponding application to the flexure of plates. R. E. Newton, USA

689. Sanks, R. L., Photoelastic analysis of stress in a dam containing a large gallery, *Proc. Amer. Soc. civ. Engrs.* **81,** Separ. no. 693, 21 pp., May 1955.

Stresses in dams are due to gravity and hydrostatic loads. The mathematical analysis is difficult or even impossible for dams with large galleries. As a first approximation, the problem was treated two-dimensionally, and models were made from BT 61-893, representing a section through the dam. The difference of principal stress is determined by the fringes, the sum by measuring the thickness changes of the model with a lateral extensometer. One model was used to freeze in centrifugal stresses, simulating gravity loading; the second was loaded at room temperature by means of an array of steel pads, simulating the hydrostatic load. Superposition of the two stress patterns obtained gives an impression of the stress distribution on combined loads.

The accuracy obtained seems to be very good, and the results could be applied for the first design of the dam.

R. G. Boiten, Holland

690. Boiten, R. G., Photo-elastic investigation of armoured models with an application to bending bars with cracks on the tensile side, *Appl. sci. Res.* (*A*) **5,** 5, 359–373, 1955.

Author describes experiments on the construction and analysis of photoelastic models equivalent to compound structural materials such as reinforced concrete. Of several polyester resins investigated, Araldite D cured at room temperature with integral aluminum reinforcement was found to be the most suitable. Plane photoelastic measurements of a rectangular beam so constructed were in close agreement with theoretical stresses. Studies were also made of the effect on the stress distribution of tensile cracks in reinforced-concrete beams, using a model with machined slits. The influence of the depth and the spacing of these slits is given in detail. W. Shelson, Canada

Rods, Beams, Cables, Machine Elements

(See also Revs. 639, 655, 656, 657, 681, 690, 713, 714, 715, 726, 727, 753)

691. Laupa, A., Siess, C. P., and Newmark, N. M., Strength in shear of reinforced concrete beams, *Univ. Ill. Engng. Sta. Bull.* **52,** 55, 1–73, Mar. 1955.

This is an analytical study with the objective of correlating the results of previous research in the field of shear and diagonal tension. Results of more than one thousand tests on reinforced-concrete beams are analyzed in an effort to establish an empirical expression for predicting the ultimate load on beams that "apparently" fail in shear. An early conclusion is that ultimate failure is never actually in shear: diagonal tension cracks reduce the effective compression zone which then fails in compression. Thus, paradoxically, ultimate failure of beams that initially fail in shear is not controlled by the maximum nominal shear stress at any section, but by the resistance moment of the reduced section. Empirical expressions are derived for this "shear-compression" moment. Results obtained by the use of these expressions compare sufficiently well with the experimental results as to suggest that the formulas may confidently be used in cases when experimental data are lacking.

An expression is first derived for beams without web reinforcement (with or without compression reinforcement) under concentrated loads. This is then modified to take care of web reinforcement and/or a T-cross section. Restrained beams and beams under uniform loading are also covered.

Some attempt is made to provide a theoretical interpretation for the empirical expressions, but even without this, it is believed that the paper represents a considerable advance in the understanding of the behavior of reinforced-concrete beams.

L. Schenker, USA

692. Tekinalp, B., Large elastic deflections of plane rods (in English, with Turkish summary), *Bull. Tech. Univ. Istanbul* **7,** 35–49, 1954.

After deriving equations of equilibrium for plane elastic rods, author introduces a perturbation method for solving problems where the deformed shape and applied force per unit length are given. Some illustrative examples are included. No error estimates or comparisons with exact solutions are given. Truesdell [*Proc. 1st Midwestern Conference on Solid Mech.*, 1953, Univ. of Illinois, 1954, pp. 52–55; see AMR **7,** Rev. 2113] has noted that such problems can always be solved by quadratures and has given an analysis of the special case of a plane cantilever loaded normally by a uniform load per unit length.

J. L. Ericksen, USA

693. Shaffer, B. W., and House, R. N., The elastic-plastic stress distribution within a wide curved bar subjected to pure bending, *J. appl. Mech.* **22,** 3, 305–310, Sept. 1955.

Analytical expressions are obtained for the radial and circumferential stress distributions within a wide curved bar made of a perfectly plastic material, subjected to a uniformly distributed bending moment. The plastic stress distributions are based on the use of the Tresca yield condition. T. H. Lin, USA

694. Kappus, R., Exact solution of a circular ring loaded with two individual forces (in German), *ZAMM* **35,** 6/7, 210–231, June/July 1955.

Paper contains exact solution of title problem, using the method of complex variable proposed first by Muskhelishvili. For easier mathematical work, problem is divided into several parts, and final results are obtained by means of superposition of partial solutions. Then comparison of classical solutions due to Timoshenko and Filon with the solution introduced is made and good agreement stated. Numerical values of stresses and strains for special ratio of the outer and inner radius of the ring are evaluated, and relations between Poisson's ratio, Young's modulus of elasticity, and deformation of the ring (flattening), important for experimental determination of these material constants, are given. V. Kopřiva, Czechoslovakia

695. Stewart, W. C., Determining bolt tension from torque applied to the nut, *Mach. Design* **27,** 11, 209–211 (Design Abtracts), Nov. 1955.

696. Fayoux, P., Equilibrium of a cable under generalized forces (in French), *Ann. Ponts Chauss.* **125,** 3, 270–282, May/June 1955.

Blondel's solution for a loaded cable is extended to include the effects of arbitrary loading and temperature variation with a nonhomogeneous cable material. H. H. Dixon, USA

697. **Johnson, R. C., Method of finite differences provides simple but flexible arithmetical techniques for cam design,** *Mach. Design* **27,** 11, 195–204, Nov. 1955.

Plates, Disks, Shells, Membranes

(See also Revs. 659, 661, 668, 674, 681, 682, 716, 717, 718, 720, 721, 722, 732, 775, 958)

698. **Plass, H. J., Jr., An approximate nonuniform bending theory and its application to the swept-plate problem,** *J. appl. Mech.* **22,** 3, 283–388, Sept. 1955.

Theory for nonuniform torsion and nonuniform bending of bars representing winglike structures is presented. The theory takes account of transverse curvature effects due to Poisson's ratio. The boundary conditions at the root of a skewed cylinder are formulated and their application to the swept-wing problem is indicated. Solutions are presented for deflections and stresses in four plates loaded in torsion at the tip and having root angles of 0°, 20°, 40°, and 60°. The deflections agree closely with experimental values. The stresses are in somewhat poorer agreement. Agreement is best about halfway down the plate.

S. Levy, USA

699. **Favre, H., and Schumann, W., Study of bending of rectangular plates with linear varying thickness and different edge conditions: application to case of hydrostatic load** (in French), *Bull. tech. Suisse Rom.* **81,** 11, 161–174, May 1955.

Solution of the governing differential equation is obtained in series form for case when two opposite sides are simply supported with any conditions at the other two sides. A complete numerical example is given for a plate which is simply supported along three sides, free at the other side, and subjected to a hydrostatic load. The deflections, moments, and stresses for the plate with linear varying thickness are compared with those for a constant thickness plate.

D. Young, USA

700. **Cadambe, V., and Kaul, R. K., Deflection of clamped plates by two-step membrane analogue,** *J. roy. aero. Soc.* **59,** 533, 358–360, May 1955.

A relaxation procedure for the solution of the clamped plate problem is presented. The crux of the method utilizes Marcus' reduction of the plate problem to two second-order membrane-type equations instead of the usual fourth-order equations. A numerical example of the dual relaxation process is presented, and results compare favorably with the Rayleigh-Ritz solution.

J. W. Mar, USA

701. **Suhara, T., On the stresses in laterally loaded plates,** *Rep. Res. Inst. appl. Mech.* **3,** 12, 180–183, Mar. 1955.

Author gives a formal solution of the three-dimensional classical elasticity equations of the problem stated in the title. The stresses are given in Fourier integral forms. The integrands are said to be solvable partly by expansion and partly by the evaluation of the residues.

G. Herrmann, USA

702. **Lihachev, V. A., Remark on the paper of A. I. Kalandiya "Bending of an elastic plate into the form of an elliptic ring"** (in Russian), *Prikl. Mat. Mekh.* **19,** 255–256, 1955.

The note points out a misstatement made by A. I. Kalandiya in a footnote on p. 702 of his paper [see AMR **8,** Rev. 1955], and indicates how certain equations in that paper can be deduced by a method of M. P. Šeremet'ev's [title source, **17,** 107–113, 1953].

I. S. Sokolnikoff, USA

703. **Nowacki, W., and Mossakowski, J., The influence surfaces of plates representing annular sectors** (in Polish, with English summary), *Arch. Mech. stos.* **5,** 237–272, 1953.

Authors investigate the bending of plates in the shape of a circular sector loaded by concentrated forces and freely supported on the edges. They solve first the problem for an infinite wedge and use this solution for an annular sector. The particular cases of shapes and loads when the deflections, moments, and shearing forces can be represented in a closed form are discussed in detail. The problem of an infinite strip is solved as a limiting case of a circular sector. In this way the solution constitutes a generalization of results obtained by other authors for infinite and semi-infinite strips.

T. Leser, USA

704. **Cleaves, H. F., The stresses in an aeolotropic circular disk,** *Quart. J. Mech. appl. Math.* (1) **8,** 59–80, Mar. 1955.

Using the method of the complex potential theory developed by Livens and Morris [*Phil. Mag.* (7) **38,** p. 153, 1947] and Morris [title source **4,** p. 248, 1951] for the solution of three-dimensional elasticity for aeolotropic bodies, a solution is given for a circular plate under various edge loadings. The results check a previous solution given by Holgate [*Proc. Camb. phil. Soc.* **40,** p. 172, 1944] for a hole in an aeolotropic plate. The solution for the plate with various edge loadings is more complicated, however. A numerical example for a circular plate of wood has been calculated using the constants determined by Holgate. A series expansion converges and results are obtained good to 0.1%.

W. P. Mason, USA

705. **Cywinska, Z., and Mossakowski, J., The influence surfaces of an orthotropic semi-infinite strip** (in Polish, with Russian and English summaries), *Arch. Mech. stos.* **6,** 33–64, 1954.

Nearly all existing solutions of an orthotropic semi-infinite strip are in terms of slowly convergent trigonometric series. In this paper, authors make a successful effort to find solutions in a closed form summing up the series. The considered strip is loaded by a concentrated force or by a moment. Authors are interested mainly in having second partial derivatives of the deflection in a closed form because this is only needed to write down moments and shearing forces. Authors found also that solutions of all the examples contain various combinations of two basic functions. They propose a close study and tabulation of these functions.

T. Leser, USA

✎706. **Novozhilov, V. V., Theory of thin shells [Teoriya tonkikh obolochek],** Leningrad, Gosud. Izdat. Sudostroit. Liter., 1951, 344 pp. $4.

This noteworthy book is written by one of the leading research workers in the field and contains much new material. Formally, the book is divided into four chapters. In the first chapter (pp. 5–85) the general theory of thin shells is presented in a very systematic manner. The author successfully develops the subject along the lines of the classical theory of elasticity by formulating equations of equilibrium, compatibility, strain energy, etc. All this is carried out in curvilinear coordinates. Throughout, the order of the approximations made is carefully pointed out. The resulting basic equations are given, using complex variables, and "complex" stress resultants and displacements are defined.

Chap. two (pp. 85–167) deals with the momentless (membrane) theory of shells, the basic equations being derived by neglecting the appropriate terms in the previously derived equations. For surfaces of revolution, the general case, subdivided into symmetrical and antisymmetrical loading, is considered. Stresses and displacements for a number of special cases are given. Consideration of membrane stresses and displacement in deter-

minate and indeterminate cylindrical shells completes the chapter. Author's procedure of treating whole classes of surfaces generated by circles, parabolas, ellipses, and hyperbolas in one formal solution appears to be particularly successfully done. The results obtained in this chapter are used in the remainder of the book.

Chap. three (pp. 167–238) is devoted to cylindrical shells. Novozhilov uses his own fourth-order equation, written in complex variables, as the basic equation for the solution of any length cylindrical shells. Later he reduces this equation for long cylindrical shells into a particularly simple form

$$T_{\varphi\varphi\varphi\varphi} + i2b^2\, T_{\xi\xi} = 0$$

where T is a "complex" stress resultant, b is a constant, and the other terms have the usual meaning. Solutions for both of the equations are given, each accompanied by an example. Here it should be noted that, in applying these methods, real and imaginary portions of "complex" stress resultants and displacements must be considered separately. Thus in the most general case, eight constants need be determined, as is the case when an eighth-order differential equation in real variables is used. Thus the reduction of the order of the basic differential equation is not as significant as may appear at first sight.

Chap. four (pp. 238–334) is concerned with surfaces of revolution. Again the basic equations are written using complex variable notation. Symmetrical and antisymmetrical loadings are considered. The latter are a particularly noteworthy addition to the literature. Solutions are elegantly carried out in general terms and asymptotic integration of the differential equations is emphasized. Specific examples are concerned with a variety of pressure vessel heads, including the case of an inward dent with toroidal transition segment. "Wind type" loads illustrate the antisymmetrical case.

The book is a welcome addition to engineering literature on the theory of shells. The treatment is mathematically rigorous and should appeal particularly to research workers to whom this book may be most highly recommended. E. P. Popov, USA

707. Reissner, E., On some aspects of the theory of thin elastic shells, *J. Boston Soc. civ. Engrs.* **42,** 2, 100–133, Apr. 1955.

Following a brief review of the statical equations of the linear membrane theory of shells in the form given by Pucher, and their reduction (in terms of Airy stress function) to a single differential equation, author illustrates a method for obtaining a solution of this differential equation by expansion in terms of a nondimensional rise of the shell; this leads to results suitable for shallow membranes. Next, by application of the principle of minimum complementary energy, a system of *suitable* stress-strain relations (involving pseudo-stress resultants) is deduced for the linear membrane theory and reduced systematically to those appropriate for shallow membranes. In the second part of the paper, Marguerre's equations for shallow shells (which include bending and nonlinear effects) are reduced to two simultaneous differential equations for an Airy stress function and for axial displacement. These equations are applied to an example of hyperbolic paraboloid shell under its own weight to study the possible buckling of such a shell and the effect of support conditions on the state of stress.

The paper, written in a lucid fashion, is expository in character and contains a number of additions to the existing literature on the subject. P. M. Naghdi, USA

708. Rüdiger, D., Extensional stresses and displacements in conoid shells (in German), *Öst. Ing.-Arch.* **9,** 1, 37–44, 1955.

A conoid shell over a rectangle is generated by a straight line moving at right angles along one side of this rectangle, sliding at the same time along a marginal curve whose plane passes through the opposite side of the rectangle and is perpendicular to it.

The extensional membrane forces are components of the so-called "tensor of longitudinal force." The latter can be calculated by solving a partial differential equation of first order.

The vector of displacement can similarly be calculated by solving another such equation. Author gives the solution of these two equations for line loads as well as for surface loading and explicit formulas for the physical components of the "tensor of longitudinal force." Notations and treatment of the problem are based on "General theory of shells" by H. Neuber [AMR 3, Rev. 1070].

Reader must be familiar with tensor analysis.

A. Werfel, Israel

709. Csonka, P., Results on shells of translation, *Acta Techn. Hung. Budapest* **10,** 1–2, 59–71, 5 figs., 7 tabs., 1955.

Analysis of shells with translation surfaces over rectangular bases; the form and loading of the shells are symmetrical relative to two vertical planes normal to each other. Paper demonstrates that the calculation of such shells, a very complicated problem in general, can be greatly simplified by an adequate choice of the shape of the shell. Condition of application of the method is that the load on the shell should be vertical and resolvable into two components, one of which is a rational whole function of one of the horizontal coordinates only, and the other a similar function of the other horizontal coordinate only. Calculation is especially simplified if the specific loading of the shell referred to the unit area of the base is constant or quadratically increasing with the distance from the planes of symmetry. Formulas are elaborated for such cases. In conclusion, a numerical example is given verifying the expediency and simple application of the method. From author's summary

710. Taylor, C. E., and Wenk, E., Jr., Analysis of stresses in the conical elements of shell structures, *Proc. second U.S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 323–331.

Based on the Love-Meissner analysis of axially symmetric thin shells, authors show that, for cones, the edge coefficients which define the relationships between edge loading and edge deformation can be expressed in terms of a number of special functions, Ω. These special Ω functions are combinations of ber_2, bei_2, or ker_2 and kei_2 functions and their derivatives and are given in graphical forms for the parametric ranges of interest.

For computing discontinuity shears and moments at shell intersections, the present method seems to be simpler than their earlier approximate solution. H. Lin, USA

711. Ambartsumyan, S. A., Applicability of some hypotheses in the theory of thin cylindrical shells (in Russian), *Izv. Akad. Nauk Otd. Tekh. Nauk* no. 5, 57–72, May 1954.

Using his own work and that of others, author writes down the equilibrium and elastic deformations for an arbitrary thin shell. For the case when only the normal loading Z is different from zero, he writes the determining eighth-order partial differential equation in the "stress" function, and gives expression for the coefficients.

The approximate solutions of shell problems are usually based on the statics hypothesis that the moments in the shell are negligible. The author considers them equal to zero and argues that the limits of applicability of the statics hypotheses are valid for the usual deformation hypotheses. Considering a circular cylindrical shell, with Poisson's ratio zero, further simplification is obtained in the determining differential equation, which does

not yet contain limitations in rise of the shell. Starting with this equation, particular cases of approximation are considered. The negligible factors are taken to be: (1) Longitudinal bending moments; (2) transverse bending moments; and (3) twisting moments.

Thus, three equations in the stress functions are written, the solutions from which are compared with the exact solution in which approximations are not made. For purpose of comparison, it is assumed that the shell is simply supported on the entire boundary. The stress function and loading Z are taken in form of trigonometric series in the usual way. Comparison of ratios of exact to approximate stress functions is shown by tables in which b/R varies from zero to 6, and $\lambda = b/a$ varies from 0.1 to 10. Here a is length of the shell, b the length on the transverse curve, R radius of the shell. Conclusion is that approximate solutions give sufficiently good results even for $b/R \geq 2$. For flat shells, the numerical comparisons confirm conclusions obtained by author in a previous paper.

For concentrated central load, the limits of applicability of above hypotheses narrow down somewhat, but this is of no practical importance if only the strength of the shell is considered.

S. Sergev, USA

712. **Galimov, K. Z., On certain problems of the theory of shells with arbitrary displacements** (in Russian), *Izv. Kazan. Filial. Akad. Nauk SSR. Ser. Fiz.-Mat. tekn. Nauk* **3,** 3-17, 1953.

The work is based on two papers by H. M. Mushtari [(1) title source **2,** 39–52, 1950; and (2) *Trudi Kazan. Him. Tehnol. Inst.* no. **13,** 1948 (not available)]. For the classification of problems for a state of initial stress, author refers the reader to (1). Setting out from results established in (2), he then proceeds to formulate the equations of equilibrium and the boundary conditions, with certain approximations, referred to Gaussian coordinates for the undeformed state and also referred to lines of curvature. No problems are solved.

L. M. Milne-Thomson, England

Buckling Problems

(See also Revs. 740, 741)

713. **Oakes, G., Analyzing column stability under varying end-load conditions (Machine design Data Sheet section),** *Mach. Design* **27,** 11, 205–208, Nov. 1955.

714. **Langhaar, H. L., Lateral buckling of asymmetrical beams,** *J. appl. Mech.* **22,** 3, 335–336, Sept. 1955.

Author derives equations for the elastic buckling strength of a beam having a single web and two parallel flanges, subjected to a uniform bending moment and axial thrust. The principal axes of inertia through the centroid are assumed to be parallel and perpendicular to the web, but the cross section need not be symmetrical about either axis. This is a special case of the more general problem of buckling under combined end load and uniform bending moment, which has been solved by J. N. Goodier [title source, **9,** A-103–A-107, June 1942] and others.

J. W. Clark, USA

715. **Csonka, P., Stability of the end-supported laterally restrained beam** (in German), *Acta Techn. Hung. Budapest* **10,** 1-2, 31–42, 7 figs., 3 tables, 1955.

The stability problem of a square cross-section beam suspended or supported at both ends and strutted by three horizontal bars is investigated in the case of a continuous, uniformly distributed load. Points of suspension or support are assumed to be at equal height above the axis of the beam: points of horizontal backing are supposed to lie on a line parallel to the beam axis above the latter. Results of investigations: Equilibrium conditions of the beam cannot become unstable unless its depth-to-width ratio exceeds 10. The height of the points of suspension or of support above the axis of the beam, and the lateral constraining structure have no effect whatever on its stability.

From author's summary

716. **Floor, W. K. G., Investigation of the post-buckling effective strain distribution in stiffened, flat, rectangular plates subjected to shear and normal loads,** *Nat. LuchtLab. Amsterdam Rap.* S 427, 26 pp., July 1953.

Strain distributions in stiffened rectangular plates under edge axial and shear loads are calculated with an assumed wave form of buckling. The distributions of the effective strain based on von Mises and Henky's criterion are shown in diagrams for various edge loading conditions to give an approximate indication of whether the elastic limit is exceeded. T. H. Lin, USA

717. **Yusuff, S., Primary instability of a panel of integral construction with unflanged stiffeners,** *J. roy. aero. Soc.* **58,** 528, 808–812, Dec. 1954.

Author uses Levy-Kroll method [AMR **3,** Rev. 867] to compute critical compressive stress-tangent modulus ratio for a range of panel slenderness ratios and reinforcement area-sheet area ratios. In the absence of local instability, panels may fail either by torsional instability of the stiffeners without cross-sectional distortion, or by pure flexure. These buckling modes are not coupled. Effect of fillets at junction of skin and stiffener is negligible.

Results of tests of eight panels show good agreement between observed and theoretically computed critical stresses. However, only two of the panels failed by torsional instability of the stiffeners; six panels failed by pure flexure.

Reviewer believes that the limited experimental verification of theory is adequate for this relatively simple case.

S. Goodman, USA

718. **Wang, C. T., Vaccaro, R. J., and De Santo, D. F., Buckling of sandwich cylinders under combined compression, torsion, and bending loads,** *J. appl. Mech.* **22,** 3, 324–328, Sept. 1955.

The differential equation applying to the problem described in the title is solved using Galerkin's method. Results agree with previous solutions in the limiting cases of axial compression alone, torsion alone, bending alone, and combined bending and axial compression. Critical stresses for combined loading are represented by the interaction formula

$$R_T = \pm [1 - (R_B + R_C)]^{1/2}$$

where R_T is the ratio of the critical torsional stress under combined loading to the buckling stress under torsion alone, and R_B and R_C are the corresponding ratios for bending and axial compression. J. W. Clark, USA

719. **Raville, M. E., Buckling of sandwich cylinders of finite length under uniform external lateral pressure,** *For. Prod. Lab., U. S. Dept. Agric.* no. 1844-B, 29 pp. + 8 figs., May 1955.

A theoretical analysis is made of the problem of the buckling of circular cylinders of sandwich construction acted upon by uniform external lateral pressure. The solution obtained is based on the assumption that the sandwich cylinder is comprised of isotropic membrane facings and an orthotropic core. The mathematical solution of the problem, which is in the form of a characteristic determinant of sixth order, is applicable to sandwich

cylinders of any length and of any core thickness. Numerical results are obtained for various values of the parameters that enter the problem, and curves are included which illustrate how the critical load varies as the values of these parameters are varied. From author's summary by T. T. Loo, USA

720. Semonian, J. W., and Peterson, J. P., An analysis of the stability and ultimate compressive strength of short sheet-stringer panels with special reference to the influence of riveted connection between sheet and stringer, *NACA TN* no. 3431, 49 pp., Mar. 1955.

Wrinkling mode of initial instability is solved by taking into account the buckle length of the plate and that of the attached flange together with the rotational stiffness offered by the attached flange. A dimension f, the distance from the stringer web to that line along which the plate and flange buckle together, is a very important part of the buckling criteria. f is less than the distance from the web centerline to the rivets but approaches this value as the rivet pitch over rivet diameter increases. The quantity f was determined experimentally for panels of 7075-T6 and 2024-T3 aluminum alloy assembled with 2117-T3 rivets. Authors believe that f is sensibly independent of the kind of rivets used.

Assuming that the failing mode is similar to the initial mode, an expression for the wrinkling mode of failure is determined from the initial mode. The resulting expression is dependent upon f. Limits of rivet pitch and rivet strength are included.

A welcome appendix for designers is a numerical example. Altogether, 14 calculations are required to determine the failing load for one panel. N. C. Costakos, USA

721. Krivetsky, A., Plasticity coefficients for the plastic buckling of plates and shells, *J. aero. Sci.* 22, 6, 432–435, June 1955.

The effect of plasticity on the buckling of plates, plate columns, or shells is considered by introducing a factor, called a plasticity coefficient τ, to allow for the departure of the stress-strain curve from the original straight elastic portion. Carpet plots are presented for the evaluation of the plasticity coefficients for five cases based on the original work of Bijlaard.

From author's summary

722. Leggett, D. M. A., The buckling of thin cylindrical shells under axial compression, *Acad. Serbe Sci. Publ. Inst. math.* 7, 47–60, 1954.

Author extends the theory of von Karman and Tsien by minimizing the strain energy with respect to four parameters instead of just two. The resulting load-over-all strain curve may contain discontinuities of the load where the energies of two possible modes of distortion, corresponding to different numbers of circumferential buckles, are equal: the mode actually adopted being that with the least energy at each strain point.

D. R. Bland, England

Joints and Joining Methods

(See also Revs. 720, 952)

723. Puzak, P. P., and Pellini, W. S., Crack arresting by overlays of notch-tough weld metal, *Weld Res. Suppl.* 577s-581s, Dec. 1955.

Crack-starter explosion tests were utilized to demonstrate the feasibility of using overlays of notch-tough welds to prevent the initiation and propagation of brittle fracture in otherwise brittle, structural mild steels. Simple bead-on-plate welds of notch-tough weld metal were deposited on the surfaces of a mild steel plate. Whereas fragmentation of the prime plate was obtained in crack-starter tests at 20 and 40 F, similar tests of plates featuring overlays of notch-tough weld metal demonstrated that brittle fractures do not propagate in overlay regions.

It is recommended that notch-tough weld deposits be considered for the protection of welded structures by application at points where brittle fractures are known to initiate, i.e., points of connections or discontinuities. The method is considered practical for application to existing or new structures.

From authors' summary

724. Sasayama, T., Masubuchi, K., and Moriguchi, S., Brief abstract of longitudinal deformation of long beam due to fillet welding, *Weld. Res. Suppl.*, pp. 581s–582s, Dec. 1955.

725. Kihara, H., and Masubuchi, K., Brief abstract of studies on the shrinkage and residual welding stress of fundamental constrained joint, *Weld. Res. Suppl.* p. 595s, Dec. 1955.

726. Love, J., Jr., and Pringle, O. A., The influence of shank area on the tensile impact strength of bolts, ASME Advance paper no. 55—A-77, 13 pp. + 12 figs., 1955.

The effect of reduction of shank cross-section area on the tensile impact strength of $^5/_{16}$-, $^3/_8$-, and $^7/_{16}$-in. steel bolts was investigated experimentally. Low carbon $^3/_8$-in. bolts with rolled threads absorbed maximum impact energy when the ratio of shank area to thread-root area was equal to 1.23. Heat-treated, medium-carbon $^3/_8$-in. bolts absorbed maximum impact energy when the ratio of shank area to thread-root area was equal to 1.17 for shanks reduced by drilling or 1.12 for shanks reduced by turning. Results for other sizes were similar. It was concluded that for maximum energy absorption the shank area should approximately equal the mean or tensile-stress area of the threads, with minor corrections to compensate for work-hardening due to thread rolling or surface oxidation resulting from heat treatment.

From authors' summary

Structures

(See also Revs. 665, 689, 691, 694, 706, 718, 739, 742, 744, 757, 956)

727. Rudmann, K., Structural design. Design of straight-member trusses and of multistory frames with parallel columns [Baustatik für die Praxis. Leitfaden für die Praxis zur Berechnung geradstabiger Balkentrager und parallelstieliger Stockwerkrahmen], Basel, Birkhauser Verlag, 1955, viii + 128 pp. SFr. 21.80.

Statically indeterminate structures are treated, the bars of which have straight center lines but varying cross sections, even along one bar. The structures dealt with are simple span girders built in at one end or at both ends, continuous multiple-supported beams, one-story or multistory frames with fixed and movable joints, one-story hall structures with vertical columns but polygonal trusses. Detailed instructions are given on the calculation of structures with elastically fixed-in beam ends or column bases and on the consideration of thermal changes and of the settlement of supports.

The calculation method applied to the problems dealt with is intermediate between the known moment-distribution method [H. Cross, "Analysis of continuous frames by distributing fixed-end moments," *Trans. Amer. Soc. civ. Engs. Pap.* no. 1793, 1930] and the fixed-point method [E. Suter-E. Traub, "Die methode der Festpunkte," Springer-Verlag, Berlin, Göttingen, Heidelberg, 1951].

At the start of the calculations, bar ends (except the hinged ones) are assumed to be rigidly fixed in; then fixed-end moments caused on bar ends by external loading are determined. These moments are summated for each joint, and balancing moments of opposite sign are distributed on the ends of bars concurring in the joint in question. Distribution is executed separately for each joint, but—contrary to the Cross method—contiguous joints are not regarded as rigidly fixed but are taken account of according to actual end-fixity conditions. Distribution and carry-over factors are, therefore, different from those applied by the Cross method; their values are determined with the aid of the moment fixed points. Moments carried over are further transferred to neighboring joints in the same way as according to the known fixed-points methods [A. Strassner, "Neuere Methoden zur Statik der Rahmentragwerke und der elastischen Bogenträger," Wilhelm Ernst und Sohn, Berlin, 1916].

The new calculation method applied in this book does not belong to the relaxation computation methods because, in the course of calculation, each joint is released *only once* and is no longer fixed. Since in the course of calculation the individual joints are balanced one after the other, simple facilities are available for the determination of moments for the different manners of loading. In this respect, the new method is a little more lengthy than the Suter method, but surplus calculatory work is amply compensated by the circumstance that the use of the sects of crossing lines, "Kreuzlinienabschnitte," introduced by W. Ritter ["Anwendungen der grafischen Statik," III. Teil: Der kontinuierliche Balken, Verlag von Albert Raustein, Zürich, 1900] has become superfluous. The elimination of this artificial concept simplifies and clarifies the calculation. On the other hand, compared with the Cross method there remains the disadvantage that the determination of stiffness and of carry-over factors is substantially more complicated here than there.

The format of the book is first class, figures are beyond reproach, lucid, perspicuous; numerical examples are instructive, easy to follow. An appendix at the end includes the most important formulas, fixed-end moments of simple-span girders with one end or both ends built in, data for beams with linear increase of cross section at the ends.

The book is aimed at meeting requirements of superstructure design and will be a handy tool for the structural analyst.

P. Csonka, Hungary

728. Brown, E. H., The energy theorems of structural analysis, *Engineering* **179,** part I, 4650, 305–308; part II, 4651, 339–342, Mar. 1955; part III, 4653, 400–403, Apr. 1955.

Nine energy theorems with unified rigorous proofs are collected in this paper. The theorems are listed in three groups according to their derivations, as follows:

A (1) Castigliano's equilibrium theorem, $\delta U/\delta\Delta = P$. (2) Theorem of virtual work. (3) Theorem of minimum total potential energy. (4) First theorem of minimum strain energy. If certain points of an elastic (nonlinear) structure are given specified displacements, the strain energy is least when the remainder of the structure is unconstrained.

B (5) Castigliano's (first) theorem for linear structures, $\delta U/\delta P = \Delta$. (6) First theorem of complimentary energy, $\delta S/\delta P = \Delta$.

C (7) Second theorem of minimum strain energy, for redundant reactions of linear elastic indeterminate structures. (8) Theorem of least work (second Castigliano theorem). (9) Second complementary energy theorem. "If the complementary energy of any initially unstressed elastic structure is expressed in terms of the n redundancies and the externally applied loads only, then each redundancy must take such a value as will make the expression a minimum."

The applicability and limitations of each theorem are carefully stated. In addition, eleven worked examples are given. This paper appears to be the only reference where all energy theorems are presented. M.-L. Pei, USA

729. Nitsiotas, G., Exact investigation of composite frames (in German), *Bauingenieur* **30, 7,** 240–244, July 1955.

Paper presents an interesting attempt to give a theoretical solution for the redistribution of interior forces in statically indeterminate frame works of prestressed concrete beams as a consequence of delayed deformations. Starting from the Dischinger equation for free deformations with the assumption that the functions determining the variation of the modulus of elasticity in time and the amount of creep and shrinkage are linearly dependent, the forces in the beams are derived in terms of creep functions. If the ratio of steel and concrete rigidities varies from beam to beam, the number of creep functions equals the product of the number μ of the beams and the number n of the statically indeterminate beam forces. The general problem leads to lengthy calculations, involving the determination of $n\mu$ eigenvalues and the solution of $n\mu$ linear equations for the integration constants. If the ratio of steel and concrete rigidities is the same for all beams, the problem is reduced to a system with n eigenvalues and n linear equations. In his concluding remarks, author mentions a greatly simplified method, proposed by Sattler. The simplification is obtained, however, by introducing rough approximations.

Reviewer believes that before application of the very complicated analytical method given in this paper is justified, more should be known on the soundness of the basic assumptions and on the quantitative data which are needed in applications.

J. F. Besseling, Holland

730. Masur, E. F., A large deflection theory of redundant structures, *Proc. second U. S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 427–446.

Author obtains an approximate large deflection solution for lateral-load, direct-load elements. The results obtained are approximate in that large deflections for the structure are not considered in the equilibrium equations or boundary conditions but are included only in a compatibility of deformation expression which the author obtains for bar elements of trussworks. Subject to these restrictions, author obtains several theorems dealing with trusses and, finally, a large-deflection ultimate load criterion for redundant trusses. This criterion is applied to a simple redundant truss and the solution obtained. It is shown in this case that the inclusion of the nonlinear effect tends to strengthen the truss. S. F. Borg, USA

731. Schwertner, A., Dynamics of girders treated by the method of elastic points, *Acta Techn. Hung. Budapest* **10,** 1–2, 155–186, 18 figs., 1 table, 1955.

The elastic point method of frame analysis is a calculatory method for solving the problem of the manifold-hyperstatic girder without simultaneous equations. There are springs at the elastic points. Spring dynamics is a simple problem which has already been solved; therefore, this method offers a new and simple solution of the problem of girder dynamics. However, springs can perform oscillations only if they possess masses. The determination of these masses is the principal subject of the paper. Examples are given for the application of the method. It is pointed out that the dynamic effect of weights dropped onto girders may also be easily calculated by the method.

From author's summary

732. Caldwell, J. B., The strength of corrugated plating for ships' bulkheads, *Quart. Trans. Instn. nav. Arch. Lond.* **97,** 3, 495–522, July 1955.

Behavior of corrugated plating under lateral load was examined theoretically and experimentally. Experiments on corrugated sheet specimens of steel and aluminum alloys show that, when instability did not occur, the behavior was adequately described by classic theories of elastic and inelastic bending. Tests to investigate the stabilization of single corrugations in bending lend support to a simple theoretical analysis. Tests to destruction on eight model corrugated bulkheads are described in terms of the analysis of simple specimens. Some conclusions are drawn concerning strength criteria for watertight bulkheads.

H. J. Grover, USA

733. Schjödt, R., Calculation of pressure of concrete on forms, *Proc. Amer. Soc. civ. Engrs.* **81,** Separ. no. 680, 16 pp., May 1955.

Formulas for calculating lateral pressure of concrete on forms are proposed by author. Factors considered include internal friction of concrete, pore-water pressure and friction between poured concrete and forms. Numerical examples show good agreement between calculated pressure and some known test results, which are considerably lower than results obtained by most empirical formulas. Reviewer believes that a much neglected field of rather complex nature has been explored by author. While everything depends on the rate of pouring and the rate of setting, the quality of concrete and form work, the least dimension of the structure, and the manner in which the concrete is placed should all have direct influence on the lateral pressure distribution. It may be mentioned that in, designing forms, the deflection under pressure, not the strength, is the criterion in most practical cases. D. H. Cheng, USA

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 672, 681, 683, 693, 721, 752, 754, 756, 763, 767, 773, 774, 958)

734. Rivlin, R. S., Further remarks on the stress-deformation relations for isotropic materials, *J. rational Mech. Anal.* **4,** 5, 681–702, Sept. 1955.

Present paper assumes that material is isotropic and the components of stress at a point of a body may be expressed as polynomials in the gradients of the displacement, velocity, acceleration, . . . , $(n-1)^{th}$ acceleration at that point. Then, the stress matrix may be expressed as a matrix polynomial in the $(n+1)$ kinematic matrixes. By the use of the Hamilton-Cayley theorem and its generalizations, it is shown in the present paper that, in special cases, the expression for the stress matrix as a matrix polynomial of arbitrary form in these kinematic matrixes can be reduced to one of closed form. K. Washizu, Japan

735. Grubin, A. N., and Lihachev, Yu. I., Analysis of the stressed state arising in the stage of large plastic deformations in stretching cylindrical specimens with ring-shaped groove (in Russian), *Zh. tekh. Fiz.* **25,** 512–528, 1955.

Analysis is given of the problem of the progressive quasistatic extension under axial force of a circular cylinder with a transverse groove cut along its surface. The problem is therefore one of rotational symmetry, and all quantities are functions of two space coordinates and a time coordinate specifying the progress of the deformation. The method is based upon a finite-strain theory of plasticity. At the outset, a power series development in an axial coordinate is assumed for the components of displacement, the coefficients in these series being unknown functions of a radial coordinate. The method proceeds through various approximations. In particular, approximations occur in connection with the above series and with the satisfaction of the boundary conditions at the progressively deforming groove. No application is made to any specific problem.

Reviewer's note: This problem is of considerable technological importance. Unfortunately, an analysis based upon an incremental theory of plasticity would be prohibitively difficult at the present time. The circumstances in which results predicted by the present analysis will be sufficiently reliable for design use are to be determined only through direct comparison with experimental data.

Courtesy of Mathematical Reviews H. G. Hopkins, England

736. Rabotnov, Iu. N., Small plastic deformations as a problem of mechanics (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 7, 97–104, July 1954.

In the belief of the reviewer, author touches upon an important point in the theory of plastic deformation of strain-hardening metallic materials. This is the general validity of the existing theories of strain hardening for small plastic deformations. On the assumption of the validity of linear relationship between the stress and strain tensors, the following main laws of plastic deformation can be checked:

(1) The constancy of volume for the plastic deformation; (2) the validity of either the maximum shear stress strain, or the octahedral shear stress-strain hardening criterion; and (3) the general validity of any criterion of strain hardening for nonmonotonically increasing types of deformation.

1. According to the author, the generally assumed validity of the constancy of volume for the plastic deformation has not been sufficiently justified. Whatever experimental evidence there is, it is not in favor of this assumption.

2. For the monotonically increasing type of deformation, the octahedral shear stress-strain hardening law has been found valid for some metals (e.g., Cr-Ni steel by the author's own tests), while the maximum shear stress-strain hardening law has been found valid for some others (e.g., copper in E. A. Davis' tests in U.S.A.). On the other hand, one Russian investigator (S. E. Ratner, 1949) could not account by either of these laws for the results on some Al-Mn alloys. [Note by the reviewer: See also Crussard, Ch., and Jaoul, B., *Rev. Métall.* **47,** p. 590, 1950.] This circumstance calls, according to the author, for a more general theory of strain hardening, including other than the quadratic stress and strain invariants, as well as anisotropic properties of the material (this is shown by the author's own experiments on predeformed and aged mild steel specimens).

3. For nonmonotonically increasing deformations, author finds, in agreement with the work done in this country, that the "deformational" theory is not applicable. However, he does not elaborate on the validity of the incremental, or the plastic flow, theory. He also reports the reduction of the shear stiffness in torsion following previous deformation in tension. This reduction is accounted for by neither the strain-hardening law nor the analysis of linear variation of stress and strain increments. In this connection, the reviewer would like to point out a more recent analysis of the above phenomena by T. H. Lin [*Proc. second U. S. nat. Congr. appl. Mech.* June 1954; Amer. Soc. Mech. Engrs., 1955, p. 461]. D. Rosenthal, USA

737. Craemer, H., Fallacies and paradoxes in the plasticity theory, *Acta Techn. Hung. Budapest* **10,** 1–2, 73–82, 9 figs., 1955.

In contradiction to the opinion of many, it is proved that the curves representing the relation between (a) stress and strain in

the tensile test, (b) load and edge strain in bending, and (c) load and deformation in bending are often confused. Plastic hinges under single loads can only be correctly interpreted if the load is assumed to be distributed along a short but finite length. Compatibility conditions are valid for any stress-strain law. In statically indeterminate structures, *full* plastification occurs simultaneously in the most stressed sections and not consecutively. Equalization of moments cannot occur before this state is reached. From author's summary

738. Sályi, I., Comments on H. Craemer's paper "Fallacies and paradoxes in the plasticity theory," *Acta Techn. Hung. Budapest* **10,** 1–2, 83–89, 5 figs., 1955.

All the relations presented in H. Craemer's paper (see preceding review) are valid within the range of small ϵ, i.e., in that of small deformations. With greater deformations, however, the formulas lose their concrete meaning and become void of any physical reality. In the latter range, naturally, they may lead to contradictions. All such contradictions can be avoided with certainty if mathematical considerations are employed only within the range where their validity is still assured with a satisfactory degree of precision. From author's summary

739. Hoff, N. J., Rapid creep in structures, *J. aero. Sci.* **22,** 10, 661–672, 700, Oct. 1955.

A survey of the effects of rapid creep upon the stress distribution in structures and of the various modes of creep failure is presented. It is proposed that rapid creep should be tolerated when aerodynamic heating causes high temperatures for only a short time in structural elements of supersonic aircraft.

From author's summary by F. K. G. Odqvist, Sweden

740. Hult, J. A. H., Critical time in creep buckling, *J. appl. Mech.* **22,** 3, p. 432, Sept. 1955.

Formula is derived for critical time from stress-strain rate law which includes elastic and steady-state creep deformations rigorously, and primary creep approximately. Inclusion of elastic effects represents generalization of earlier theory. Reviewer considers this important because it often affects critical time materially. N. J. Hoff, USA

741. Ketter, R. L., Stability of beam columns above the elastic limit, *Proc. Amer. Soc. civ. Engrs.* **81,** Separ. no. 692, 27 pp., May 1955.

Assuming one-parameter deflection curves, the collapse load for beam columns is calculated by equating the changes of internal and external moment at a specified station, due to virtual curvature changes. Strain hardening is neglected. Specimen stiffnesses are deduced from graphs of a previous note [AMR **7,** Rev. 3566]. P. Cicala, Italy

Failure, Mechanics of Solid State

(See also Revs. 723, 725, 749, 755, 763, 767, 771)

742. Lundberg, B., Fatigue life of airplane structures, *J. aero. Sci.* **22,** 6, 349–413, June 1955.

Author emphasizes that the primary difficulty in the problem of fatigue in airplane structures is the large scatter in test results associated with all aspects of the problem. Due to this scatter, it is proposed that work on the fatigue problem be concerned not with prediction of fatigue life but with the reduction of fatal fatigue failures to a low failure rate. The failure rate could be based upon the combination of a "limit" life and its corresponding "limit" probability level and should be agreed upon internationally. Using these ideas of probability and failure rate, author presents a survey of the various aspects of the fatigue of complete airplane structures, together with numerical applications. A general formula, developed for the calculation of cumulative damage, is used for the preparation of charts for the calculation of the fatigue life as a function of the most important design and fatigue parameters. A method by Weibull for the determination of *S-N* curves at various probability levels is discussed and applied. The paper deals mainly with fixed-wing transport aircraft, although methods can be applied to all types of airplanes. B. E. Gatewood, USA

743. Beckert, E., Stress calculation according to Bach or according to fatigue life (in German), *Maschinenbau-Technik* **4,** 6, 285–291, June 1955.

Author propagandizes for the use of fatigue data in design as being more accurate than the Bach method. Some simple illustrative examples are given. A. Isaksson, Sweden

744. Duncan, W. J., The Comet and design against fatigue, *Engineering* **179,** 4647, 196–200, Feb. 1955.

Author first discusses the technical aspects of the accidents to the two Comet airplanes. He explains that they were caused by failure of the pressure cabin by fatigue under repeated application of the normal working pressure and other minor fluctuating loads. The skin stresses were too high and the windows were of a shape that produced severe stress raisers. Author then outlines several of the known concepts relating to fatigue of metals and their implication in design. Topics discussed are combined stresses, cumulative damage, size effect, stress raisers, and residual stresses. Principles are given for proper design of structures to withstand fatigue loads. A. Yorgiadis, USA

745. Kawamoto, M., and Nishioka, K., Safe stress range for deformation due to fatigue, *Trans. ASME* **77,** 5, 631–634, July 1955.

See AMR **8,** Rev. 1007.

746. Higuchi, M., On the dualism in the fracture theory, *Rep. Res. Inst. appl. Mech.* **3,** 12, 155–169, Mar. 1955.

Mode of fracture of plastics is predicted by analysis of elastic and plastic elongations that take place under varying strain rates, of boundaries surrounding microscopic, irregular-shaped domains. Boundaries are assumed to be composed of a chain of sites of some minor constituents, including flaws, of weaker strength than those of the base material. Microfracture is assumed to occur on the attainment of a limiting local tensile elongation regardless of nature of external force system. Variation of breaking strength and of fracture energy of a number of plastics is analyzed. A linear relationship is found to exist between fracture strength and strain rate on a log-log plot. Fracture energy vs. strain-rate curve is found to have a positive slope below and negative above the transition strain-rate value, with the fracture energy converging to the same value at the transition strain rate. It is suggested that a similar fracture mechanism may be applicable to metals. D. Kececioglu, USA

747. Waterhouse, R. B., Fretting corrosion, *Instn. mech. Engrs. Preprint,* 10 pp., 1955.

A very general paper on fretting corrosion with a brief analysis of the subject after literature study. F. Macks, USA

748. Anonymous, Fretting and fretting corrosion, *Lubrication* **41,** 8, 96 pp., Aug. 1955.

Material Test Techniques

749. Corten, H. T., Dimoff, T., and Dolan, T. J., An appraisal of the Prot method of fatigue testing, *Proc. ASTM* **54,** 875–894, 1954.

The effectiveness of fatigue testing under progressively increasing load level, usually referred to as the Prot method, in determining the endurance limit of metals is investigated. The test results suggest that this method is reasonably accurate for ferrous metals without "coaxing" due to strain aging; where coaxing is significant, the endurance limit obtained in a Prot test is too high. The testing procedure appears to be unsatisfactory for light metals as well as for all metals in the presence of time-dependent effects, such as corrosion or creep.

A. M. Freudenthal, USA

750. Seemann, H. J., and Schmauch, H., Determination of elastic constants of metallic materials by aid of measurement of sound velocity. Application for the case of castings (in French), *Rev. Métall.* **52,** 8, 621–628, Aug. 1955.

Mechanical Properties of Specific Materials

(See also Rev. 691)

751. Comstock, G. F., Titanium in iron and steel, 1955, New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd., xii + 294 pp.

Book is the third in the new series of Alloys of Iron Monographs. It is a critical review and correlation of the world literature by the foremost authority on the subject.

Effects of titanium on mechanical properties of carbon and alloy steels, cast irons, stainless steels, and complex heat-resisting alloys are discussed at length. Also, effects of titanium as a deoxidizer and on nitrogen and sulfur in steel are described.

Data are arranged in convenient tables. Appropriate graphs and other illustrations are shown. An extensive bibliography is included. Volume well maintains the high standard of the earlier volumes.

R. J. Anderson, USA

752. Betteridge, W., Alloys for use at high temperatures, *Brit. J. appl. Phys.* **6,** 9, 301–306, Sept. 1955.

The properties necessary in metallic materials intended for service at high temperatures are outlined and the characteristics of nickel-base alloys for such applications are particularly described. The materials are dealt with in two classes: those primarily of value because of their resistance to oxidation, and those characterized by high strength at elevated temperatures.

From author's summary

753. Carlson, C. R., Selection and application of spring materials, ASME Advance paper no. 55—A-76, 3 pp. + 9 tables + 1 fig., 1955.

This paper is intended to aid engineers and designers to select a proper spring material for each spring design quickly, easily, and accurately. This purpose is accomplished by grouping similar materials into tables and by listing selection and application data in a short summary to simplify the task of selection from among the forty spring materials in general use.

From author's summary

754. Zhukov, A. M., Strength and plasticity of D16T alloy in a complex state of stress (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 6, 61–70, June 1954.

755. Pellini, W. S., Sandoz, G., and Bishop, H. F., Notch ductility of nodular iron, *Trans. Amer. Soc. Metals* **46,** 418–445, 1954.

The notch ductility transition characteristics of various types of nodular irons were determined by Charpy V tests. Impact and explosion tests of castings containing a sharp notch were used to evaluate the significance of the Charpy data to service and to establish comparisons with cast and rolled steels subjected to the same tests. It is established that the transition range of Charpy energy curves is the important criterion which correlates with fracture resistance in the impact and explosion tests. The energy values associated with fibrous fractures of Charpy V specimens of nodular irons are not significant in this respect. It is demonstrated that optimum notch toughness is obtained with fully ferritized material of low silicon and phosphorus contents. Comparison with cast and rolled steels indicates that fully ferritized nodular irons of present commercial composition are less resistant to brittle fracture than the steels.

From authors' summary

756. Sherby, O. D., Goldberg, A., and Dorn, J. E., Effect of prestrain histories on the creep and tensile properties of aluminum, *Trans. Amer. Soc. Metals* **46,** 681–700, 1954.

Creep and tensile data were correlated with the various substructures developed during creep testing of annealed and cold-worked pure aluminum. Whereas the creep strength and tensile strength of annealed specimens increased as the structure became more highly polygonized, the tensile strength decreased and the creep resistance increased following creep and increased polygonization of cold-worked structures. Tentative explanations are offered in an attempt to account for the experimental results. An equilibrium structure and strength are approached during creep straining which are dependent only on the creep stress, independent of the temperature and previous history of the metal.

From authors' summary

757. Sutton, H., Some aspects of modern aircraft materials, *J. roy. aero. Soc.* **59,** 535, 451–477, July 1955.

758. Smith, L. P., Silicones as engineering materials, *Engineering* **179,** 4665, 789–792, June 1955.

759. Warde, J. M., and Johnson, J. R., Recent developments in the technology of ceramic materials for nuclear energy service, *J. Franklin Inst.* **260,** 6, 455–466, Dec. 1955.

760. Steinitz, R., Cermets—new high-temperature materials, *Jet Propulsion* **25,** 7, 326–330, July 1955.

During the past decade, new materials have been developed on the basis of combinations of metals with refractory compounds which to some extent fill the gap between so-called superalloys and ceramics. The specific characteristics of these cermets and the general requirements for high-temperature material are discussed. Cermets are produced by powder metallurgy, and this special fabrication method results in unique properties. The high-temperature properties of some semicommercial cermets are compared with standard superalloys. The present-day shortcomings of cermets and the direction in which future development work will have to be directed are discussed.

From author's summary

761. Bäcker, M., New materials (plastics) (in Polish), *Przegl. tech.* **75,** 2, 50–54, Feb. 1954.

762. **Carson, W. G., and Robertson, E. N., Synthetic fabric-acrylic resin impregnates for aircraft canopy edge attachments,** *Aero. Engng. Rev.* **14,** 3, 44–49, Mar. 1955.

763. **Findley, W. N., and Khosla, G., Application of the superposition principle and theories of mechanical equation of state, strain, and time hardening to creep of plastics under changing loads,** *J. appl. Phys.* **26,** 7, 821-832, July 1955.

The superposition principle and the theories of mechanical equation of state, strain, and time-hardening were applied to the creep under changing loads of four unfilled thermoplastics—polyethylene; monochlorotrifluoroethylene (Kel-F), crystalline; polyvinyl chloride (Geon-404), annealed; and polystyrene.

A comparison of the theoretical results predicted by these theories with the experimental results indicated the accuracy with which these theories described the creep behavior under changing loads for the above materials. From authors' summary

764. **Werren, F., Supplement to mechanical properties of plastic laminates,** *For. Prod. Lab. Rep., U. S. Dept. Agric.*, no. 1820-B, 9 pp., 4 tabs., 6 figs., Sept. 1955.

This report presents the results of tension, compression, bending, bolt-bearing, and interlaminar shear tests of two parallel-laminated epoxide laminates. The laminates were tested after normal conditioning, with some tests also made after wet conditioning. Tension, compression, and bending data are presented both in tables and as average curves. Limited bolt-bearing and interlaminar shear data are presented in a table.

From author's summary

765. **Shimozuru, D., A note on the elasticity of marble,** *Bull. Earthq. Res. Inst., Tokyo Univ.* **33,** part 1, 73–77, Mar. 1955.

766. **Shimozuru, D., and Murai, I., Elasticity of marble with special reference to its elastic aeolotropy,** *Bull. Earthq. Res. Inst., Tokyo Univ.* **33,** part 1, 65–72, Mar. 1955.

767. **Hunt, F. V., Elastic-plastic instability caused by the size effect and its influence on rubbing wear,** *J. appl. Phys.* **26,** 7, 850–856, July 1955.

The yield strength of many materials is much higher for minute specimens than for bulk samples. The region around the point of highest shear stress in a solid undergoing deformation by a small spherical indenter can be regarded as such a minute specimen which may be "protected" by the size effect against plastic yielding if it is small enough. Formulating the effective yield strength and the elastic stress under the indenter in terms of a common parameter provides a basis for assessing the influence of size scale on the plastic yield threshold. Four size categories are identified, including a critical case for which a small change of loading may cause a discontinuous transition from the elastic to the plastic regime throughout the region of contact, and another, more frequently encountered, in which the supportable preyield elastic stress is materially enhanced. The latter effect may exert an important influence on the rate of rubbing wear since it can make available a wider range of loading for which a low wear rate prevails. Reported wear tests on steel riders and on sapphire phonograph styli confirm these predictions qualitatively.

From author's summary

768. **Ludewig, H., Trelon—a new polyamide yarn** (in Polish), *Przegl. tech.* **75,** 6, 215–216, June 1954.

Mechanics of Forming and Cutting

(See also Rev. 952)

769. **Perow, W. A., and Brand, W. J., Fine grinding of ores [Feinmahlen der Erze]** (translation from Russian), Berlin, VEB Verlag Technik, 1954, 190 pp.

After describing the theoretical principles of grinding in general, and the wet fine grinding of ores in tube mills in particular, author gives the calculation of the output and efficiency of laboratory and industrial mills. He compares the results of laboratory experiments with industrial experiences. He describes in particular the performance of ball mills of cylindric and conic form with different methods of feeding and discharging. He gives the theory of ball mills, the energy consumption, and deals with the foundation, the mounting, operation, and repair of the mills and with the organization of repairs. He compares the different types of mills, gives advice on choosing the right type corresponding to the material, to the output, and to the fineness of grinding.

A separate chapter deals with the designing of grinding plants and gives practical instructions for operation.

F. Hinsenkamp, Hungary

770. **Hahn, R. S., The effect of wheel-work conformity in precision grinding,** *Trans. ASME* **77,** 8, 1325–1329, Nov. 1955.

The conformity between wheel and work is defined by the curvature difference between the wheel and the trochoidal surface of the work against which it is pressed in grinding. According to experiments, the rate of metal removal varies as the 0.18 power of this curvature difference. The latter depends not only on the ratio of the work/wheel radii but also on the surface speed ratio and the mode of grinding (external, internal, surface).

This explains why the rate of removal is smaller for internal than for external grinding. It is also consistent with the fact that at low work speed the rate is proportional to the work speed and independent of wheel speed, and vice versa, at high work speed. A theory is proposed which partly agrees with experimental results. H. R. Fehling, England

771. **Shiba, J., Abrasion of metals (part 1). General investigation in abrasive phenomena,** *Bull. Fac. Engng. Yokohama Nat. Univ.* **4,** 1–9, Mar. 1955.

772. **Weiner, J. H., Shear-plane temperature distribution in orthogonal cutting,** *Trans. ASME* **77,** 8, 1331–1341, Nov. 1955.

An analytical solution is presented for the temperature distribution along the shear plane in orthogonal cutting. The principal assumption made is that the heat transferred by conduction in the direction of motion of the workpiece or chip may be neglected. The nature of the error involved in this assumption is investigated. The over-all fraction of shear-plane heat transferred into the workpiece is also computed and compared with previous results.

From author's summary by E. G. Loewen, USA

773. **Thomsen, E. G., and Frisch, J., Stresses and strains in cold-extruding 2S-O aluminum,** *Trans. ASME* **77,** 8, 1343–1353, Nov. 1955.

A split cylindrical billet of 2S-O aluminum was extruded at room temperature. The extrusion was interrupted at frequent steps to photograph the deformed pattern of an initially square inscribed grid. By an original technique named "visioplasticity" all quantities of interest were computed from these observed deformed grid patterns. Remarkable similarities were found between the results and those previously obtained by the

senior author and others [AMR 8, Rev. 1671] for lead. Perhaps even more remarkable are the similarities between the observed results and theoretical results for a perfectly plastic solid in plane strain obtained by Lee [see Prager and Hodge, AMR 4, Rev. 4444 (pp. 120–121)]. This would indicate that the great theoretical differences between plane strain and rotationally symmetrical problems may not be of practical importance in plasticity problems involving large strains.

P. G. Hodge, Jr., USA

774. Danese, G., The cold-rolling of iron plate. Calculation method simplified (in Italian), *Ingegnere* **29,** 3, 263–275, Mar. 1955.

The pressures, friction, pull, and stresses acting upon the plate as it passes between the rolls are analyzed. Some factors disregarded in previous derivations are considered here. The equation of stress along the contact surface is established. The importance of friction between plate and rollers is discussed. Results are shown to compare favorably with those of von Karman, Trinks, and Tselikov, and others. J. P. Vidosic, USA

775. Oehler, G., Problems in connection with cold forming of thick plates with special regard to punching and bending (in German), *Schiffbautech. Gesellsch. Jahrbuch* **48,** 130–137, 1954.

776. Szombathy, E., Use of pneumatic length-measuring devices with special reference to their use in increasing production (in Hungarian), *Mérés és Automatika* **3,** 6, 161–169, June 1955.

In modern production, the quality of products is greatly influenced by the accuracy of measurements. In the interest of cost reduction and of increase in production, the automatization of the measurements is very important. The application of pneumatic principles in measurement techniques unites both these goals. This paper points out a few important domains of applications and discusses theoretical and practical aspects of the problems. From author's summary by T. P. Torda, USA

Hydraulics; Cavitation; Transport

(See also Revs. 786, 796, 815, 950, 963, 964, 966)

777. Leliavsky, S., Irrigation and hydraulic design, Vol. I, New York, Macmillan Co.; London, Chapman & Hall, Ltd., 1955, xiii + 492 pp. $25.

This is the first volume of a luxurious monograph, an important handbook for designers. Author is a distinguished Egyptian engineer and professor and son of a famous Russian hydraulic engineer, N. S. Leliavsky (1853–1905). This volume covers basic chapters: Percolation of water as a structural design factor; tail erosion; hydraulics of canals and rivers. Author explains many modern ideas and methods, among them some by Russian and Indian scientists, little known to our readers; e.g., Khosla's percolation trajectories, Pavlovsky upthrust curves. Flow net construction is explained by graphical and experimental methods, such as Forchheimer's especially emphasized circle method, Pavlovsky's electric analogy, magnetic field. Reviewer would mention the excellent American method by H. Foster (1944).

Percolation in earth dams and embankments is treated very thoroughly. Tail erosion is based on author's experience on Egyptian dams and models. Now and then he reviews some specific problems, as conformal transformation, theory of elliptic functions, of standing wave. Chapter on canals and rivers is very interesting and inspiring; it was published earlier in a British journal. Author reveals several controversial problems: velocity distribution curve, the "friction fallacy," inadequacy of hydraulic radius. Historical sketch on velocity scale by J. Komlosi (1941, 1942) would add much glamor to this old problem. Interesting idea for open channels, originated by A. van Rinsum (1936), could also be mentioned. One more fallacy could be added, concerning dimension of the roughness coefficient in Manning formula.

On pp. 388 ff, author explains graphical determination of the so-called Boussinesq factor, as actually involved Coriolis factor is related by a doubtful empirical equation. The term "Gunder's amendment" cannot be justified, as different coefficients were used by many authors long before 1943.

Text and abundant figures are of great value for hydraulic engineers. Technical completion is on high level. The spelling of several names must be improved (Rümelin instead of Rennelin, Einwachter-Enwachter, Humphreys-Humphrey, Brünings-Brunnings, Woltman-Woltmann, John R. Freeman-T. R. Freeman, Nikuradse-Nikuradze; unusual spelling of Darcy as d'Arcy).

S. Kolupaila, USA

778. Gibrat, R., The operation of tidal power stations in view of the limitations imposed on the speed of discharge variations (in French), *Houille blanche* **10,** special A, 275–282, May-June 1955.

The use of a tidal power-station site at the same time for both producing power and for other purposes imposes limits on the speed of the discharge variations, which for the Rance have been fixed by the Administration at $\pm$ 3 m^3/sec^2. A convenient solution of this limitation consists in starting up (or stopping) each power station unit one after the other.

In starting up or shutting down with one way flow it is seen that, by having the condition of maximum available efficiency (as determined from previous studies) at the beginning of flow, there is little difference between the power produced by the two methods of operation of each unit, sudden and gradual (the difference for the Rance is less than three in a thousand). On the other hand, the head loss in the last unit started up and the first unit shut down is negligible.

With the flow active in both directions the problem involves the gate maneuvers for the turbines, and the gates themselves. Very complicated in theory, this can be solved for the Rance by numerical calculations based on four distinct hypotheses, all beginning from the operation of the units at maximum power. Comparison of the power results reveals a clear advantage for two solutions: one in which all the turbines work at maximum power at the beginning and then are cut off at a certain stage of the filling; in the other, all the turbines work all the time but according to a pattern which integrates the effect of the preceding partial shutdown.

But this clear advantage and the small difference between the two solutions derived show at the same time the interest and the limitations of the theoretical research to be done.

From author's summary

779. Gruat, J., Studies of constant cross-section surge tanks with the aid of electric analogies (in French), *C. R. Acad. Sci. Paris* **240,** 22, 2120–2122, June 1955.

Author reports some new results of an electronic analog computer devised by him for studying surge phenomena. Computer was described in an earlier article and is based on well-known electromechanical correspondences, the quadratic law of resistance being obtained by a push-pull of electronic tubes with positive polarization of grids. G. Evangelisti, Italy

780. **Schultz-Grunow, F., Stability of a liquid surface by periodically switched-off earth gravity** (in German), *VDI-Forschungsheft* **21**, 1, 27–28, 1955.

When a glass of water is submitted to a periodic vertical motion, the relative acceleration periodically changes, and it may be that this acceleration periodically becomes zero; the system is at this moment without weight. There is no reason in this case for the surface of the liquid to remain coherent.

Author presents the theory of this motion: He finds that the oscillations of the surface of the liquid are solutions of the Mathieu differential equation. The destruction of the surface of the liquid corresponds with the zones of instability of the Mathieu equation. He also presents experiments showing the correspondence of the practical results with the theory.

L. T. Tison, Belgium

781. **Ellis, A. T., Production of accelerated cavitation damage by an acoustic field in a cylindrical cavity,** *J. acoust. Soc. Amer.* **27**, 5, 913–921, Sept. 1955.

Most studies of cavitation damage have been made with apparatus in which the damaged body is subject to accelerations and forces which may lead to the premature removal of materials from surfaces which have been weakened by cavitation. This effect is avoided in the method described in the present paper, which consists in making the surface under investigation a fixed boundary in a cylindrical vessel containing liquid in which an ultrasonic field is set up using a barium titanate transducer in the form of a ring. The frequencies of the normal modes of the experimental system are calculated and they are found to be in good agreement with the experimental values.

Experiments have been carried out with a number of metals and alloys and with Stellite, pyrex, and fused quartz; in the majority of the experiments, water with an air atmosphere was used, but some experiments were carried out with nickel using toluene with a helium atmosphere. After exposure to the ultrasonic field for various times, the specimens were examined by microphotography and x-ray techniques. The results indicate that the stresses due to cavitation were at least of the order of 50,000 psi. It is also possible that metals undergo plastic deformation which leads to fatigue and failure of the specimens. The experiments with a helium atmosphere show that severe damage occurs even when a chemically inert atmosphere is used.

R. M. Davies, Wales

782. **Shalnev, K. K., Structure of the cavitation zone** (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 5, 119–146, May 1954.

The object of this investigation was to determine the structure of the cavitation zone behind a round profile. The investigations included visual and photographic observations of the cavitation as it appeared, developed, and disappeared. In particular, the following were studied: (1) Period of formation and floating of individual cavities; (2) influence of the pressure on structure of the cavitation, when velocity and Reynolds number remained constant; (3) influence of the velocity on the structure of cavitation; and (4) influence of the stage of advancement on structure of cavitation.

Although results appear to have no particular practical value, the investigation is a valuable contribution to our knowledge and understanding of the cavitation phenomenon.

W. B. Palijenko, Canada

783. **Knapp, R. T., Cavitation (Forum on Hydraulics),** *Mech. Engng., N.Y.* **77**, 12, 1063–1064, Dec. 1955.

784. **Parkinson, J. B., NACA model investigations of seaplanes in waves,** *NACA TN* 3419, 28 pp., July 1955.

Paper describes the two towing tanks and associated equipment such as models, towing gear, and instrumentation, at the Langley Aeronautical Laboratory. Main emphasis is laid on the facilities for rough water landing and taxiing tests. Typical results of general application for the effects of hull and wave proportions and hull form on the dynamics of the model and also the results on take-off resistance are shown and the trends discussed.

A. F. W. Langford, Australia

785. **Larned, A. T., Hydraulic prime movers (Forum on Hydraulics),** *Mech. Engng., N.Y.* **77**, 12, 1058–1059, Dec. 1955.

Incompressible Flow: Laminar; Viscous

(See also Revs. 784, 817, 830, 833, 835, 849, 850, 902, 927, 933, 934, 940, 954, 957, 963)

786. Birkhoff, G., Hydrodynamics, a study in logic, fact and similitude, 1st Dover ed., New York, Dover Publications, Inc., 1955, xiii + 186 pp. $1.75 (paperbound).

See AMR **3**, Rev. 2692.

787. **Stuart, J. T., A solution of the Navier-Stokes and energy equations illustrating the response of skin friction and temperature of an infinite plate thermometer to fluctuations in the stream velocity,** *Proc. roy. Soc. Lond. (A)* **231**, 1184, 116–130, July 1955.

An exact solution of the Navier-Stokes equations is presented for the special case of a two-dimensional, incompressible, constant property flow for which the velocity components are independent of the x coordinate. The system treated is an infinite plane wall having a porous surface with a constant section velocity through the surface. The velocity field far away from the plate is periodic. The solution is presented in terms of the skin-friction phase angle and amplitude, showing generally that the skin friction has a phase lead over the velocity fluctuations and increasing amplitude with frequency.

In the second part of the paper, the incompressible constant property form of the energy equation with dissipation is treated for the case of an adiabatic wall in conjunction with the unsteady velocity field of the first part of the paper. The results show that the mean wall temperature increases with frequency by virtue of the increase in dissipation of kinetic energy to thermal energy.

R. M. Drake, Jr., USA

788. **Pearcey, T., and McHugh, B., Calculation of viscous flow around spheres at low Reynolds numbers,** *Phil. Mag.* (7) **46**, 378, 783–794, July 1955.

The characteristics of the flow pattern around a smooth sphere moving at uniform velocities through a viscous medium are determined in this paper through use of a solution of Oseen's approximations of the hydrodynamic equation for a viscous medium. Computations are carried out for the relatively low Reynolds numbers of 1, 4, and 10. This study is important in providing basic information in the understanding of the drag forces existing on objects moving through viscous fluids at low Reynolds numbers and in further attacking problems relating to the interaction of bodies passing through viscous fluids. The results are presented in nondimensional form for distances up to 10 sphere radii away from the moving sphere. An important conclusion of this study is that there are rudimentary evidences of a wake at Reynolds numbers even as low as 1, and a wake is definitely present at Reynolds numbers of 4 and 10.

Reviewer believes this paper supersedes other efforts to calculate the characteristics of the flow pattern around spheres at low Reynolds numbers. Paper would have been of somewhat more value if, in addition to the nondimensional graph, presentations made from streamlined patterns had also been drawn.

D. W. Pritchard, USA

789. Ericksen, J. L., On the uniqueness of ideal gas flows with given streamline patterns, *Bull. tech. Univ. Istanbul* **6, 6,** 1–5, 1953.

A neat proof that, if vortex and source-sink flows are excluded, there are at most three flows having a given streamline pattern, the fluid being nonviscous and thermally nonconducting.

The method consists in showing that if $\phi = m^2 - 1$, where m is the Mach number of the flow, then ϕ satisfies a cubic equation.

J. C. Cooke, British Malaya

790. Hämmerlin, G., On the eigenvalue problem of three-dimensional instability of laminar boundary layers on concave walls (in German), *J. rational Mech. Anal.* **4, 2,** 279–321, Mar. 1955.

Author recalculates the critical value of the Görtler parameter $\lambda = (U_0\vartheta/\nu)(\vartheta/r)^{1/2}$ as a function of the wave number $\alpha\vartheta$, where U_0 is the main-stream speed, ϑ the momentum thickness, r the radius of the surface, and ν the kinematic viscosity. For the basic mode of oscillation, a critical λ equal to about 0.3 is obtained, which is roughly in accord with the value obtained by Görtler [*Nachr. Ges. Wiss. Göttingen* **2,** 1, 1940] and by di Prima [*Quart. appl. Math.* **13,** 1, 55, Apr. 1955]. Another calculation by Meskyn [*Proc. roy. Soc. Lond.* (*A*) **203,** 253, 1950; see AMR **4,** Rev. 1278], gave a critical λ several times higher, but which was closer to the analogous Taylor number for flow between concentric cylinders, when the inner cylinder rotates and the outer is at rest [Taylor, *Phil. Trans.* (*A*) **223,** 289, 1923]. Reviewer believes that Meskyn's work gives a higher mode of oscillation than the basic one, and, correspondingly, that the lowest critical Taylor number (or Görtler parameter) is higher in the cylinder case than in the concave-wall case because of the added constraint of the second wall in the former problem. Author also obtains the result that the critical λ occurs for $\alpha\vartheta = 0$, a result out of accord with earlier work, which gives a finite value of $\alpha\vartheta$ for the critical λ.

Earlier works (Görtler, Meskyn) and recently di Prima found that λ tends to infinity as $\alpha\vartheta$ tends to zero. In order to examine the solution for small $\alpha\vartheta$, author proceeds as follows: He lets $\alpha\vartheta$ tend to zero in the differential equations, and assumes that $\lambda\alpha\vartheta$ tends to zero also. Since λ also appears in the boundary conditions at the edge of the layer, the equations and boundary conditions yield a value of λ as $\alpha\vartheta$ tends to zero, and this is in fact the lowest or critical λ. Furthermore, the solution near $\alpha\vartheta = 0$ tends quite nicely into the solution (obtained by other means) for larger $\alpha\vartheta$. Author does not prove that no solution exists for which λ tends to infinity as $\alpha\vartheta$ tends to zero. Bearing in mind this point, reviewer would like to offer comment on two points. First, it is known from Meskyn's work that many modes of oscillation exist, having different λ for a given $\alpha\vartheta$. If λ takes a finite value as $\alpha\vartheta$ tends to zero for the lowest mode of oscillation, it seems natural to expect similar behavior for higher modes. But the author's method yields a single value of λ as $\alpha\vartheta$ tends to zero, and if this gives the lowest mode, why is the behavior of other modes different? Secondly, in the analogous case of flow between rotating cylinders, extremely good agreement was obtained between the theoretical and experimental Taylor and wave numbers of the critical disturbance. In particular, a nonzero wave number was predicted theoretically and verified experimentally, without taking into account the flow curvature ϑ/γ except through the Taylor number. According to the author, it is necessary to include terms of order ϑ/γ in the equations for flow past a concave wall in order to obtain a nonzero critical wave number. If this is correct, it is surprisingly different from the rotating-cylinder case. The reviewer feels that the author's result is not fully convincing as long as these two points remain unexplained. J. T. Stuart, England

791. Kiselev, A. A., On the unsteady flow of a viscous fluid with external forces (in Russian), *Dokladi Akad. Nauk SSSR* (*N.S.*) **100,** 5, 871–874, 1955 (translated from Russian by M. D. Friedman, 572 California St., Newtonville, Mass., 6 pp.)

Consideration is given to the nonlinear Navier-Stokes equations with an external force which would be applicable to unsteady incompressible flow within a rotating cylinder. A general solution is constructed using finite differences. Existence and uniqueness are discussed with the aid of a related integral equation.

Problem and symbols are not clearly defined, if at all. Hence, paper is extremely difficult to comprehend.

S. Ostrach, USA

792. Rouse, H., and Siao, T.-T., Form drag of composite surfaces, *Proc. second U. S. nat. Congr. appl. Mech.,* June 1954; Amer. Soc. mech. Engrs., 1955, 743–749.

Useful data are given on drag coefficients for groupings of plane bodies arranged in a plane normal to the direction of flow. Objects tested are: rings varying in solidity from thin strip to solid disk, grids made of concentric rings varying in number from one to infinity and varying in solidity, lattices of orthogonal strips varying in number of strips and in solidity, and groups of square plates. Conclusion is that form drag of a body may be increased by slotting it, provided the body is of the class investigated here.

R. A. Burton, USA

793. Miesse, C. C., The effect of ambient pressure oscillations on the disintegration and dispersion of a liquid jet, *Jet Propulsion* **25,** 10, 525–530, 534, 15 figs, 9 refs., Oct. 1955.

The effect of ambient pressure oscillations on the disintegration and dispersion of a liquid jet was investigated by imposing a high-intensity acoustic field on the jet. The experimental set is described whereby a transversal and a longitudinal oscillating sound pressure can be produced. The effect of sound pressure, directed perpendicularly to the stream, was to disperse the droplets in a diverging sinusoidal configuration; the effect of an axial cavity resonance directed parallel to the stream was to coalesce the droplets, as a consequence of the velocity variation of successive fluid particles. Each of these effects was analyzed theoretically; it was found that the magnitude of each effect decreased with an increase either in the velocity of the stream or in the frequency of the imposed oscillation.

The theoretical and experimental work, while treated with considerable generality, is specifically directed to the clearing up of phenomena in a liquid-propellant rocket combustion chamber. The following main conclusions have been drawn: (1) Ambient pressure oscillations, either normal to or parallel to the axis of liquid jet, tend to decrease the length of the solid stream and have a decided effect on the dispersion pattern of the jet. (2) It can be shown both experimentally and theoretically that the magnitude of this effect decreases as either the pressure drop across the orifice or the frequency of the pressure oscillations is increased. (3) Transverse pressure waves aid considerably in the mixing of parallel streams. (4) The coalescence of the droplets, which result from axial pressure oscillations in the chamber, can lead to unstable combustion if the steady-state flow velocity is less than a certain critical value. For the thorough understand-

ing of this paper, a study of the previous paper of the same author is recommended ["Correlation of experimental data on the disintegration of liquid jets," *Indust. Engng. Chem.* **47**, 9, 1690–1701, Sept. 1955]. K. J. De Juhasz, Germany

794. van der Leeden, P., Nio, L. D., and Suratman, P. C., The velocity of free falling droplets, *Appl. sci. Res.* (*A*) **5**, 5, 338–348, 1955.

The velocity of free fall of droplets of various liquids and for several sizes has been determined as a function of the height of fall. The results are discussed in connection with Reynolds and Weber numbers. The drag coefficients found are independent of Re and a unique function of We. The $C_d - We$ curve shows a discontinuity at $We = 5$.

From authors' summary

795. Dityakin, Y. F., On stability and disintegration into drops of a liquid jet of elliptic section (in Russian), *Izv. Akad. Nauk SSSR. Otd. tekh. Nauk* no. 10, 124–130, 1954.

The jet, of ideal liquid surrounded by fluid, is disturbed in such a way that the surface varies periodically with time and with distance from the orifice. Author uses elliptic coordinates and Mathieu functions to find the critical frequency at which the jet becomes unstable. A dimensionless graph of frequency against pitch shows that the more nearly circular the section, the finer the drops. L. M. Milne-Thomson, England

796. Tan, H. S., On source and vortex of fluctuating strength travelling beneath a free surface, *Quart. appl. Math.* **13**, 3, 314–317, Oct. 1955.

797. Kravtchenko, J., and McNown, J. S., Seiche in rectangular ports, *Quart. appl. Math.* **13**, 19–26, Apr. 1955.

The authors' problem is to find a velocity potential of the form $\cosh k(z + h)F(x, y)$, where $(\nabla^2 + k^2)F(x, y) = 0$ in the rectangle $(0 \leqq x \leqq a, 0 \leqq y \leqq b)$; the normal gradient $\partial F/\partial n$ is to vanish at the boundary except on the segment $(\alpha \leqq x \leqq \beta, y = 0)$, where $\partial F/\partial n = A$, a constant. This segment represents the entrance to the port, and it is stated that there is experimental evidence for the approximate validity of the boundary condition. It is shown that, in principle, a solution can be obtained in the form $F = f + u$ where f satisfies the boundary conditions but not the equation of continuity, while $\partial u/\partial n = 0$ along the whole boundary. The function u can be expanded in an adequately convergent double series of eigenfunctions. When the forcing frequency coincides with a resonance of the closed rectangle, this series requires modification. F. Ursell, England

Compressible Flow, Gas Dynamics

(See also Revs. 786, 818, 819, 820, 829, 830, 862, 869, 870, 899, 900, 907, 957)

798. Goldstine, H. H., and von Neumann, J., Blast wave calculation, *Comm. pure appl. Math.* **8**, 2, 327–353, May 1955.

Spherically symmetrical propagation and decay of shock wave generated by very strong point-source explosion in perfect gas are studied theoretically. Flow behind shock is treated as nonviscous, nonconducting, and isentropic, and the shock itself is treated as an adiabatic discontinuity satisfying the Rankine-Hugoniot equations. In nomenclature, r should be Eulerian radius, R Lagrangian radius, rather than the reverse.

Major part of paper concerns scheme for replacing system of differential equations behind shock by difference equations, and for putting them in a form suitable for programming on a digital computer with speed and accuracy.

Numerical results for case with initial pressure ratio across shock of 100:1 are displayed. Included are (1) shock-pressure ratio and shock speed versus shock radius; (2) time of shock arrival versus shock radius; (3) pressure distribution behind shock versus radius, at various times; (4) pressure distribution behind shock versus time at various fixed locations.

A. H. Shapiro, USA

799. Brun, E. A., and Plan, M., Thermal measurements in condensation shocks (in French), *C. R. Acad. Sci. Paris* **239**, 19, 1183–1185, Nov. 1954.

The method described allows determination of local stagnation temperature θ_∞ in supersonic flow behind a condensation shock. The temperature of an insulated wall will be measured once with dried air θ_{fd} and once with wet air θ_{fw}. With the known stagnation temperature in the inlet θ_0, then

$$\theta_\infty = \theta_0 + \theta_{fw} - \theta_{fd}$$

Author notes that this method indicates effects of humidity which are so small that they are not shown by schlieren system.

S. F. Erdmann, Holland

800. Kawamura, T., Contribution of the change of entropy to the directions of spines of shock polar, *J. phys. Soc. Japan* **9**, 396–400, 1954.

Expressions are derived for the initial slopes in the hodograph plane of streamlines of a uniform stream which passes through a curved two-dimensional shock. The slopes are found as a function of the free-stream Mach number and the angle of inclination of the shock, first taking account of the entropy variation behind the shock and then neglecting this entropy change. Results are plotted as a function of the angle of inclination for the case of free-stream Mach number equal to 2, and a considerable deviation is found in the range in which the initial slope of the streamlines in the hodograph plane is 90°. Author concludes that, in order to study the flow characteristics of the subsonic flow region behind a curved shock, it is necessary to take proper account of the entropy variation. P. Chiarulli, USA

801. Nonweiler, T., Theoretical supersonic drag of non-lifting infinite-span wings swept behind the Mach lines, *Aero. Res. Counc. Lond. Rep. Mem.* no. 2795, 22 pp., 1954.

This paper is concerned with the wave drag of a constant-chord sweptback wing with straight subsonic leading edge and constant symmetrical wing sections at zero incidence. For the case of infinite span, author obtains an elegant and compact expression for the wave drag, which is reminiscent of the formula for the induced drag in lifting-line theory. It is shown that, for given sweepback, the wave drag can be represented as the product of two terms which are, respectively, independent of the flight speed and of the wing section. Various examples are given and the dependence of the drag on certain parameters is discussed in detail.

A. Robinson, Canada

802. Turner, T. R., Effects of sweep on the maximum-lift characteristics of four aspect-ratio-4 wings at transonic speeds, *NACA TN* 3468, 25 pp., July 1955.

An investigation at transonic speeds has been made in the Langley high-speed 7- by 10-ft tunnel to determine the effect of wing sweep on the maximum-lift characteristics of a series of wings having aspect ratio of 4, taper ratio of 0.6, and NACA 65A006 airfoil sections. The Mach number varied from 0.61 to 1.20 with the Reynolds number varying from 380,000 to 460,000.

Maximum lift coefficients increased with increased sweep at the lower Mach numbers but decreased with increased sweep at

the higher Mach numbers so that there was less variation of the maximum lift coefficient with Mach number as the sweep was increased. From author's summary

803. Michael, W. H., Jr., Flow studies on flat-plate delta wings at supersonic speed, *NACA TN* 3472, 40 pp., July 1955.

An experimental study has been made to investigate some aspects of the nature of the flow around delta wings. Vapor-screen, pressure-distribution, and ink-flow studies were made at a Mach number of 1.9 on a series of semispan delta-wing models with slender wedge airfoil sections and very sharp leading edges. The models had semiapex angles ranging from 5° to 31.75°.

Separated regions of vorticity existed along the chords of all the wings in the series tested. Concentrated vortex cores were found only on wings of very small semiapex angles. For wings with medium and large semiapex angles, the separated vorticity was concentrated in a region extending over the outboard part of the span and lying close to the wing upper surface.

The results show that theoretical aerodynamic calculations, such as those in *NACA TN* 3430, utilizing a single, separated vortex pair above the wing upper surface to represent the separated vorticity, can be applied at supersonic speeds for very slender wings. From author's summary

804. Vaglio-Laurin, R., and Ness, N., Analytical and numerical methods for hyperbolic conical flows, *Polyt. Inst. Brooklyn Rep.* no. 273, 77 pp. + 19 figs., July 1955.

Hyperbolic portion of nonaxially symmetric conical flow field is analyzed by method of characteristics. Equations are presented for numerical calculation although no numerical examples are given. Also, Ferri's method of linearized characteristics is extended to conical problems. One result of this extension is that curvature of attached shock at leading edge can be related to curvature of conical shape. K. C. Harder, USA

805. Stewartson, K., On the motion of a flat plate at high speed in a viscous compressible fluid. II. Steady motion, *J. aero. Sci.* **22,** 5, 303–309, May 1955.

The theory of the steady flow of a viscous compressible fluid past a flat plate at high Mach number due to Lees and Probstein [AMR **6,** Rev. 3146] is extended by a more complete discussion of the flow in the inviscid layer between the shock wave and the boundary layer. It is shown that similar solutions exist in this layer, analogously to those found by Li and Nagamatsu [AMR **6,** Rev. 1972] in the boundary layer, and that the two may be joined to give, allowing one minor assumption, a full account of the flow. It is shown that the boundary-layer equations may be reduced to those for an incompressible fluid and that the von Karman-Pohlhausen method describes the flow in it with good accuracy. The tangent wedge approximation for the pressure on the plate, used by Lees and his collaborators, is found to be in deficit by 10% for air. Finally, it is shown that the theory for weak interaction cannot be extended further without a complete knowledge of the flow. C.-C. Chang, USA

806. Stewartson, K., On the motion of a flat plate at high speed in a viscous compressible fluid. I. Impulsive motion, *Proc. Camb. phil. Soc.* **51,** part 1, 202–219, Jan. 1955.

The effect of giving an infinite plate, immersed in a compressible fluid, an impulsive velocity U in its own plane is examined on the assumption that $M = U/a_0 \gg 1$, where a_0 is the velocity of sound in the undisturbed fluid. To begin with, the flow field is very complex but, very soon after the motion has begun, a shock wave moves at right angles to the plate, gradually weakening and eventually degenerating into a sound wave. Attention is concentrated on the flow field when there is a distinct strong shock wave, it being assumed, and justified a posteriori, that the flow behind it may be divided into an inviscid region and a boundary-layer region. The equation of the shock wave and a complete description of the flow behind it is found when $(M^4\nu_0/a_0^2t) \gg 1 \gg \nu_0/a_0^2t \log M$, where ν_0 is the coefficient of kinematic viscosity of the undisturbed air. An approximate method is also given to enable the solution to be joined up to Van Dyke's work which is valid when $(M^4\nu_0/a_0^2t) << 1$.

From author's summary by C.-C. Chang, USA

807. Pai, S. I., Two-dimensional supersonic shear flow around a corner, *Proc. second U. S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 637–642.

The first-order correction to irrotational Prandtl-Meyer flow is derived for small vorticity of the oncoming (inviscid) stream. Analytic solutions and numerical examples are given. It is shown in passing that a parallel shearing flow is turned by a corner into a nonparallel flow. C. M. Ablow, USA

808. Sterbentz, W. H., and Davids, J., Amplitude of supersonic diffuser flow pulsations, *NACA TN* 3572, 23 pp., Oct. 1955.

Helmholtz resonator approach to ramjet diffuser pulsation employed in *NACA RM* E51C27 is extended to consider nonlinear damping terms present at finite amplitude. Resulting differential equation is integrated stepwise by assuming a line segment approximation to the pressure-recovery versus mass-flow curve. Variation of pressure amplitude with mean mass flow and pressure variation with time for given mean mass flow are obtained and compared with experimental results for two cases. Theory also predicts that stable mass-flow range of diffuser may be increased by decreasing combustion-chamber volume.

Reviewer has shown in *NACA RM* L52A18 and L53G28 that ramjet pressure fluctuations in absence of burning are governed by quasi-one-dimensional traveling waves, and ramjet acts like an organ pipe partially closed at one end. It was shown therein that, when oscillation wave length was much longer than ramjet length, Helmholtz resonator analysis was valid to determine approximate frequency and average (since resonator analysis predicts constant amplitude everywhere in chamber) combustion-chamber pressure amplitude.

Reviewer also argued and presented experimental verification that resonator concept, since dependent on over-all geometry of ramjet, was inapplicable for determination of initial instability, which he showed to be dependent on local conditions in vicinity of inlet. Consequently, reviewer questions value of resonator prediction for increasing stable range. R. L. Trimpi, USA

809. Rosen, J., The design and calibration of a variable Mach number nozzle, *J. aero. Sci.* **22,** 7, 484–490, July 1955.

A method of designing a symmetric Mach number nozzle with adjustable flexible walls has been developed. The continuous variation of the Mach number is controlled by one single screw mechanism, which, in a defined manner, simultaneously affects the coordinates and derivatives of the inlet and throat curvatures up to the inflection point. The flexible part of the curvature between the inflection point and the test section consists of a spring, with linear thickness distribution, rigidly clamped at the level of the test section (150 × 150 mm² = 5.9 in. × 5.9 in.). Due to small curvatures and a suitable choice of the linear thickness distribution of the flexible part of the wall, a good Mach number distribution (±0.015 or less) has been realized within the whole Mach number range from 1.5 to 2.5.

From author's summary by H. J. Allen, USA

810. **Yur'ev, I. M., On the calculation of nozzles** (in Russian), *Prikl. Mat. Mekh.* **19,** 103–105, 1955.

Author solves the equations of potential flow in the neighborhood of the throat of a plane channel or axially symmetrical nozzle by means of a transonic approximation. Using coordinates based on the center of the throat section, (x,y) in plane flow, and (x,r) in axially symmetrical flow, the reduced potential function is assumed to be a quadratic function of x, the coordinate measured downstream along the axis of symmetry. The coefficient of x^2 is constant, but other coefficients depending on y (or r) are determined from the equations of motion. Simple expressions for the velocity distribution and shape of linear are derived. Maurice Holt, England

811. **Skripkin, V. A., On transonic flow of a plane gas jet from a nozzle with parallel walls** (in Russian), *Prikl. Mat. Mekh.* 19, 89–98, 1955.

Author considers oscillatory motion of a slightly supersonic jet into an atmosphere of slightly lower pressure. Entropy changes are ignored and equations of motion simplified by use of transonic approximation. Analytical solutions are developed in parts of the expanding half of the first wave of the jet. Solutions in the simple wave regions centered on the edges of the nozzle unit can be written down immediately and determine conditions on the upstream sides of the quadrilateral in which these centered waves interact. The motion in the region of interaction is then found in the hodograph plane by solving the Euler-Poisson equation by Riemann's method. In the general case, the analytical solution is terminated at the downstream vertex of the quadrilateral, but, in the special case of a sonic jet, the analytical solution is extended to cover the whole of the expanding half of the wave.

Author incorrectly states that the flow pattern in the second, compressive half of the first wave is a mirror image of that in the expanding half. In fact, as is well known, this compression is achieved through shocks passing through nodal points in the free streamlines. Only the part of this motion ahead of the shock can be determined by reflection of the expanding flow pattern. Maurice Holt, England

812. **Sterbentz, W. H., and Evvard, J. C., Criterions for prediction and control of ram-jet flow pulsations,** *NACA TN* 3506, 60 pp., Aug. 1955.

Theoretical and experimental study of stability of flow through supersonic diffusers with and without subsequent combustion. Problem is simplified by assuming pressure waves can be treated as in a Helmholtz resonator. Results indicate that, without combustion, disturbances will be resonantly amplified if the variation of pressure recovery with mass flow has a positive slope of sufficient magnitude. With combustion, the rate of heat addition markedly alters the resonant properties. Methods of minimizing flow pulsations are discussed.

This is a republication of a paper originally issued in 1951. Subsequent work has refined understanding of this problem. For instance, Trimpi, in *NACA RM* L53G28, shows Helmholtz-resonator method to be useful only in approximating frequency and amplitude of oscillations and that the method should not be used to determine stability criteria. W. F. Davis, USA

813. **Moeckel, W. E., Estimation of inlet lip forces at subsonic and supersonic speeds,** *NACA TN* 3457, 12 pp., June 1955.

Author discusses effects of inlet lip thickness on inlet performance as functions of mass flow for subsonic and supersonic speeds. At subsonic speeds, pressure-recovery losses and additive drag are shown to decrease linearly with increasing lip frontal area if maximum suction force is realized. At supersonic speeds, inlet drag increases linearly with lip frontal area at full mass flow; for reduced mass flow, some reduction in total drag is possible with lips of moderate thickness, but this decreases with increase in flight speed.

Analysis is one-dimensional; thus suction force is independent of lip shape, and analysis assumes cowling cross section straight and aligned with the stream flow. W. H. Pell, USA

814. **Stehling, K. R., and Foster, R. W., Liquid jet atomization by a sonic nitrogen stream,** *Jet Propulsion* **24,** 6, 384–385, Nov.-Dec. 1954.

As part of a research project on a rocket thrust chamber, a study was undertaken on the atomization and penetration of liquid (water) streams injected at various velocities and angles into a sonic gas stream. From authors' summary

Wave Motion in Fluids

(See also Revs. 668, 780, 786, 815, 900)

815. **Bratfisch, A. E., and Cartwright, K. O., Water hammer calculations and test results—Owens Gorge power plant penstocks,** ASME Advance paper no. 55—A-71, 8 pp., 1955.

Water-hammer calculations were made for the three penstocks of the Owens Gorge Power Plants to determine proper governor and relief-valve time settings. Test results show good agreement with the calculations. Computing methods, complete test results, and an analysis of these results are included in this paper. A brief discussion of the operation of the surge chambers for these plants is also included. From authors' summary

816. **Kerr, S. L., Water hammer (Forum on Hydraulics),** *Mech. Engng., N.Y.* **77,** 12, 1059–1060, Dec. 1955.

Turbulence, Boundary Layer, etc.

(See also Revs. 787, 790, 805, 828, 833, 840, 894, 901, 902, 924, 925, 926, 959, 961, 964)

817. **Stewartson, K., The asymptotic boundary layer on a circular cylinder in axial incompressible flow,** *Quart. appl. Math.* **13,** 2, 113–122, July 1955.

Paper deals with laminar boundary on a long cylinder where thickness of the layer is of same order as the radius a of the cylinder. Consideration is given to main stream velocities proportional to x^m, x being the axial distance U from the leading edge. Procedure used involves the solution of equations of motion for the boundary layer in terms of a stream function ψ, which are then simplified under certain assumptions for large $\psi = 2\nu x/Ua^2$, ν being the kinematic viscosity. From two iterations the writer is led to assume a certain type of series for ψ, the first two terms of which are those obtained by Oseen for large x. The main result is that the effect of the curvature is to increase the skin friction at large distances downstream where the boundary layer is thicker than the radius of the cylinder. This is found to correlate well with the results of Seban and Bond [AMR 5, Rev. 1161], who also found an increase in skin friction (over that for a flat plate) for regions near the leading edge. Author infers that the effect of curvature at distances where the boundary layer is of the same order as the radius is to delay separation.

This paper is of theoretical interest, but one naturally wonders if a laminar boundary layer can be found in those regions on a long cylinder for which this theory is thought to be applicable. Perhaps some idea of this could be obtained if numerical results were included. In most cases of practical concern the boundary

layer becomes turbulent. A semiempirical approach to the effect of transverse curvature on turbulent shear stress has been given by L. Landweber [*David W. Taylor Mod. Basin Rep.* 689, Mar. 1949]. J. P. Breslin, USA

818. Flügge-Lotz, I., and Johnson, A. F., Laminar compressible boundary layer along a curved insulated surface, *J. aero. Sci.* **22,** 7, 445–454, 490, July 1955.

The von Karman-Pohlhausen integral method is applied with a fifth power polynomial for velocity profile and a tenth power one for density. The additional velocity boundary condition, beyond those used by Pohlhausen for the low speed case [*ZAMM* (1) **1,** 1921], is in agreement with the y-derivative of the momentum equation at the wall.

Like Pohlhausen's case, the results are more accurate for accelerating than for decelerating flow. S. Corrsin, USA

819. Dunn, D. W., and Lin, C. C., On the stability of the laminar boundary layer in a compressible fluid, *J. aero. Sci.* **22,** 7, 455–477, July 1955.

A more complete theory is developed, modifying certain general conclusions in the earlier treatment of Lees and Lin [*NACA TN* 1115, 1946; *NACA Rep.* 876, 1947, formerly *TN* 1360]. The main changes are the inclusion of three-dimensional disturbances and the demonstration that, in many cases, the stability characteristics depend on temperature fluctuations.

At high supersonic free-stream velocities, the relation for the characteristic values is found to be dependent on temperature fluctuations. For subsonic and slightly supersonic velocities, temperature fluctuations are unimportant.

For the boundary-layer flow over a flat surface, three-dimensional disturbances are found to be of little importance at low subsonic free-stream Mach numbers, except possibly under conditions of extreme surface cooling. As the Mach number increases, at a Mach number between one and two, they begin to play the leading role in many cases of practical interest.

At supersonic free-stream Mach numbers, the boundary layer can never be completely stabilized by cooling, with respect to all three-dimensional disturbances.

From authors' summary by S. Corrsin, USA

820. Morduchow, M., and Grape, R. G., Separation, stability, and other properties of compressible laminar boundary layer with pressure gradient and heat transfer, *NACA TN* 3296, 45 pp., May 1955.

This investigation uses an approximate solution of the boundary-layer equations by an integral method and the two-dimensional stability theory of Lees and Lin [*NACA TN* 1115, 1946, and *NACA Rep.* 876, 1947]. Constant wall temperature, a linear viscosity-temperature relation, and Prandtl number of unity are assumed. Illustrative numerical examples are given throughout.

The total effect of wall temperature on the skin-friction and heat-transfer coefficients is found to be composed of two parts. The first is a pressure-gradient effect, for which cooling tends to diminish the two coefficients in a favorable pressure gradient and increase them in an adverse one. The second is an effect associated with the viscosity-temperature relation, for which cooling ordinarily tends to increase both coefficients. The direction of the combined effect of cooling and favorable pressure gradient depends on the Mach number in some cases.

A simple and ordinarily sufficiently accurate method is derived for determining the separation point in an adverse pressure gradient. Cooling is found to delay separation generally. For a fixed ratio of wall temperature to adiabatic wall temperature, an increase of Mach number moves the separation point upstream, while for a fixed ratio of wall temperature to free-stream temperature it generally has a less unfavorable effect and may even move the separation point downstream.

Calculations of the minimum critical Reynolds number and the wall temperature required for complete stabilization indicate that a favorable pressure gradient and cooling are normally both stabilizing. The reviewer notes that Low [AMR **7,** Rev. 2541] has also investigated the effect of pressure gradient on the cooling required for complete stabilization. D. W. Dunn, USA

821. Chernyi, G. G., Boundary layer with a surface of discontinuity. Flow along a plate with fluid seeping through its surface (in Russian), *Dokladi Akad. Nauk SSSR* (*N.S.*) **100,** 5, 867–870, 1955 (translated from Russian by M. D. Friedman, 572 California St., Newtonville, Mass., 7 pp.).

Author studies the boundary-layer flow along a plane wall with porous surface through which fluid of different physicochemical properties is introduced. The seepage ratio C was assumed small so that on both sides of the surface of discontinuity separating the two fluids (region 1 outer flow, region 2 inner flow), Prandtl's boundary-layer approximation holds valid. This insures the discontinuity surface to lie well inside the boundary layer. The equations of continuity, momentum, and energy then lead to usual boundary-layer equations for both regions

$$\rho(uu_x + vu_y) = (\mu u_y)_y,\ (\rho u)_x + (\rho v)_y = 0$$

$$\rho(uT_x + vT_y) = (\mu T_y)_y/Pr + m^2\mu\, uy^2 \qquad [1]$$

where Pr denotes constant Prandtl number, and $m^2 = \nabla^2/c_pT_0$. The solutions for two regions are joined along the surface of discontinuity, which in turn is to be determined.

It is clear that u, v can be solved independently of T, after which T follows by quadrature. On introducing $\rho u = \phi'(\zeta)$, $\rho v = (\zeta\phi' - \phi)/2x^{1/2}$, $T = T(\zeta)$, $\zeta = y/x^{1/2}$; and further transforming to new function $\omega = \mu\, u\zeta$ and independent variable u, [1] becomes

$$\omega_{uu} + \rho\mu\, u/2\omega = 0$$

$$T_{uu} + (1 - Pr)\omega_u Tu/\omega + m^2Pr = 0 \qquad [2]$$

where $\rho\mu$ is to be taken constant ($\rho_1\mu_1 = 1$, $\rho_2\mu_2 = K^2$). $\omega(u)$ and u^* (velocity on surface of discontinuity) are then uniquely determined by five conditions

$$\omega_2{}'(0) = C/2,\quad \omega(1) = 0 \qquad [3]$$

$$\omega_1{}'(u^*) = 0,\quad \omega_2{}'(u^*) = 0,\quad \omega_1(u^*) = \omega_2(u^*)$$

after which ζ follows from integral $\zeta = \int \omega^{-1}\alpha u$.

It is observed that solution $\omega(u)$ so obtained involves two parameters, C and K. Results are obtained by numerical integration, and plotted as a set of one parameter (K) family of curves correlating the skin friction coefficient $C_f(Re_x)^{1/2}$ and seepage ratio C. For fixed values of K, a tendency of decreasing friction coefficient with increasing seepage is noted.

Curve $K = 1$ should be checked with Schlichting's result [H. Schlichting and K. Bussman, *Schr. Deutch. Akad. Luftfahrtforsch.* **73,** no. 2, 1943]. H. S. Tan, USA

822. Poppleton, E. D., Boundary-layer control for high lift by suction at the leading edge of a 40 deg swept-back wing, *Aero. Res. Counc. Lond. Rep. Mem.* no. 2897, 38 pp., Oct. 1951, published 1955.

Wind-tunnel tests have been made on a 40° sweptback wing, 10% thick with constant chord and an aspect ratio of 4.6. Boundary-layer control was applied along the whole leading edge, and a comparison was made between the effects of distributed suction

and suction through a slot. A 45% Fowler flap was used in some tests.

The over-all effect of the two systems was similar, giving an increase in $C_{L\max}$ by increasing the stalling incidence and making the wing statically stable up to the stall, when there was a severe loss of lift. At $R = 1.29 \times 10^6$, a gain in $C_{L\max}$ of 0.5 was obtained when $C_Q = 0.0023$ with distributed suction over the first 2.5% chord, while a C_Q of 0.0074 was needed to obtain the same increment with a slot (0.18% chord wide at 2.5% chord). A maximum value of $C_{L\max}$ of 1.95 was obtained by both methods with flaps down at $\alpha = 25°$.

For full-scale application, the suction required for distributed suction (8.2 lb/sq in.) is much higher than for the slot, but it may be possible to reduce this by grading the porosity in a chordwise direction or by dividing the leading edge into a number of separate spanwise compartments. With the slot, the quantity required is higher but the over-all power is about the same. A reduction in power may be possible in this case, by improving the shape of the slot.

As a means of producing high maximum lift on swept wings, both these methods have the disadvantage of requiring variable-incidence wings, if the full gain in $C_{L\max}$ is to be used. Either method can be used as a means of preventing tip stalling as an alternative to nose flaps or leading-edge slots.

From author's summary

823. Persh, J., The behavior of the boundary layer in the region of transition from laminar to turbulent flow, *J. aero. Sci.* 22, 6, 443–444, June 1955.

Note in Readers' Forum.

824. Carrow, D. D., A note on the boundary layer and stalling characteristics of aerofoils, *Aero. Res. Counc. Lond. curr. Pap.* 174, 20 pp., Oct. 1950, published 1954.

A qualitative explanation is suggested for the changes which occur in the stalling characteristics of airfoils as the Reynolds number is increased. This explanation is based on the variation, with Reynolds number, in the state of the boundary layer along the upper surface at just below the stalling incidence.

From author's summary

825. Persen, L. N., Some remarks on von Karman's equation of isotropic turbulence, *Proc. second U. S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 667–675.

Author assumes, following von Karman, that the double and triple correlation function $f(r, t)$ and $h(r, t)$ vary only with $r(a + bt)^{-1/2}$, and he explores the conditions for compatibility of this assumption with the dynamical equation connecting f and h. One of two alternative conditions is that the kinetic energy of the turbulence is proportional to $(a + bt)^{-1/2}$. This is not exactly in accordance with measurements (and neither is the "self-preservation" assumption), but reviewer believes it to be generally accepted as approximately valid. However, author rejects it, and the remainder of the paper is devoted to examining the consequences of the alternative set of conditions. These consequences can be brought into agreement with observations of $f(r, t)$ as a function of r. G. K. Batchelor, England

826. Birkhoff, G., Fourier synthesis of homogeneous turbulence, *Comm. pure appl. Math.* 7, 1, 19–44, Feb. 1954.

Author considers homogeneous isotropic turbulent velocity fields, as represented, in the space X at a given time t, by Gaussian (= normal) random vector fields; the main result is that to each absolutely continuous energy spectrum (= pure band spectrum, without concentration of energy on lines) corresponds essentially one Gaussian random vector field; the proof is done by the construction of a Gaussian (completely additive) measure on the function space of all incompressible vector fields $\mathbf{u}(\mathbf{x})$. Part I develops heuristic considerations based on the superposition of Boussinesq's flows (sinusoidal shear flows); part II contains the highly technical methods of measure theory leading to the construction of the measure; one of the interesting steps is the definition of a measure on a Hilbert space. The measure is invariant for any translation or rotation in the space X; it is also reversible, i.e., invariant for the transformation $\mathbf{x} \to \mathbf{x}$, $\mathbf{u} \to -\mathbf{u}$. Part III is devoted to the consequences of the theoretical results; one important implication of the reversibility is that, in the evolution of turbulence, the correlation tensor must satisfy the heat equation. This not being the experimental case, it shows that the Gaussian hypothesis might be only approximately fulfilled in a real turbulence.

J. Kampé de Fériet, France

827. Millsaps, K., The Obukhoff spectrum of homogeneous isotropic turbulence, *J. aero. Sci.* 22, 7, p. 511, July 1955.

828. Bass, J., Space and time correlations in a turbulent fluid. II. Space and time spectrum, *Univ. California Publ. Statist.* 2, 85–98, 1954.

[For part I, see title source 2, 55–83, 1954.] Author states that this paper is an introduction to the study of the general spectrum of turbulence, considering velocity correlations at two different instants of time. Only the geometrical aspects of the question are investigated; the dynamical properties of the spectrum are not considered. The analysis is quite thorough and should, therefore, provide useful background for further work. Some limited comparison with experiments is given.

C. C. Lin, USA

829. Burgers, J. M., A model for one-dimensional compressible turbulence with two sets of characteristics, *Proc. k. Ned. Akad. Wet.* (*B*) 58, 1 (part I 1–8) (part II, 9–18), 1955.

Author continues his previous studies of one-dimensional turbulence by using again the one-dimensional equation of motion, but with the addition of a term $-a^2\rho\partial\,\rho/\partial\,x$ on the right, and the one-dimensional compressible continuity equation. The equations form a system with two sets of characteristics similar to those appearing in the motion of a gas; consideration of the statistical problem given in previous studies of the incompressible one-dimensional equation is not undertaken. The form of the solution is found to be sensitive to the form of the viscous term. If the correct one-dimensional equation of motion is used, with the pressure gradient term, along with the continuity equation, three sets of characteristics are found. Morever, the characteristics are not straight lines. The behavior of shocks traveling in the two media described by the two sets of equations is discussed.

A. M. Kuethe, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 784, 792, 801, 802, 803, 804, 822, 824, 848, 849, 850, 851, 854, 860, 863, 864, 865, 867, 868, 870, 872, 941, 945, 959, 962)

830. Timman, R., Theory of thin airfoils in unsteady incompressible or compressible flow (in French), Fluid Mech. Meeting (Marseille 1952), *Publ. sci. tech. Min. Air, France* no. 296, 285–327, 1955.

Fluid is considered to be perfect and the influence of viscosity is taken into account by means of the Kutta-Joukowsky condition, with the surface of discontinuity (wake). Linearization is achieved by means of developing the potential into a power series

of a small parameter, determined by the geometry of the airfoil. Three methods are considered: velocity potential, acceleration potential, and integral equations (method of vortexes). For an incompressible fluid, all three methods furnish the same results, but each of them has particular merits in generalization to more complicated problems. The method of the conformal representation is the most useful in considering more complicated geometric forms. The method of the acceleration potential admits generalization to the subsonic flow, but this is not easy. The method of integral equations seems to be unadaptable for seeking exact solutions. Some numerical approximations can be obtained. Paper is a valuable addition to the field.

M. Z. Krzywoblocki, USA

831. Pindar, A. C. S., and Collingbourne, J. R., Pressure plotting and balance measurements in the high-speed wind tunnel on a half-model of a 90-deg-apex delta wing with fuselage, *Aero. Res. Counc. Rep. Mem.* no. 2844, 54 pp., Sept. 1949, published 1954.

Tests were made at a Reynolds number of 1.8×10^6 and Mach numbers up to 0.93. The wing tip was cropped to a taper of 1/7 and the wing section was RAE 102, symmetrical, 10% thickness/chord at 35% chord.

Form drag is highly localized near the root at low speed. Above $M = 0.88$, rearward movement of the strong shock causes a rapid rise of drag at all sections.

Spanwise loading at low incidence is close to potential theory for wing without body up to $M = 0.9$. A tip stall occurs at $M \geq 0.9$ for $\alpha = 3.65°$ and at $M \geq 0.8$ for $\alpha = 7.7°$, and causes a nose-down moment. Over-all lift slope at low C_L's increases to a maximum at about $M = 0.89$, then falls off with signs of a recovery at $M = 0.92$.

Local aerodynamic centers at low C_L agree with potential theory for wing alone at low speeds, but move backwards beyond $M = 0.8$. The over-all aerodynamic center for the wing moves back about 10% mean chord by $M = 0.92$.

There is a loss of elevon power for angles up to $-5°$ above $M = 0.92$, as found on a complete model at lower Reynolds number. From authors' summary

832. Pearcey, H. H., and Faber, M. E., Detailed observations made at high incidences and at high subsonic Mach numbers on Goldstein 1442/1547 aerofoil, *Aero. Res. Counc. Lond. Rep. Mem.* no. 2849, 52 pp., Nov. 1950, published 1955.

833. Hurley, D. G., and Ward, G. F., Experiments on the effects of air jets and surface roughness on the boundary layer near the nose on an NACA 64A006 aerofoil, *Aero. Res. consult. Comm. aero. Res. Lab. Melbourne, Austral., Note* 128, 21 pp., 22 figs., Sept. 1953.

Experiments have been made on the nose of an NACA 64A006 airfoil to compare the improvements that can be obtained in its stalling characteristics using either a row of air jets or surface roughness located near the leading edge. The test Reynolds number was 3.85 million. From authors' summary

834. Naylor, C. H., Notes on the induced drag of a wing-tail combination, *Aero. Res. Counc. Lond. Rep. Mem.* no. 2528, 10 pp., July 1946, published 1954.

An expression has been derived for the factor to be applied to ideal induced drag to allow for wing-tail interference. This factor is primarily dependent on the wing-tail lift and span ratios. It is of the order of 1.1 for a normal aircraft when the tailplane carries 10% of the weight of the aircraft, and can reach unexpectedly large values at high speed.

Charts, generalized curves, and sufficient information are included to permit rapid evaluation of the factor for any particular case. From author's summary

835. Santangelo, G., Unsteady and steady correctly banked turns of airplanes with any type of propulsion unit (in Italian), *Aerotecnica* **34,** 1, 3–12, Feb. 1954.

After some preliminary considerations, the equations of a general turn are stated and, as a particular case, the ones of a correct turn. Because of the difficulty of a direct solution it is advisable to investigate the turn by taking into consideration only the equations of the c.g. motion and adding to them a third equation which states the "maneuver pattern" of the actual turn, instead of the equations of motion about the body axes. Steady and unsteady turns in incompressible flow are systematically investigated, and a procedure to extend the same investigation to the compressible flow is indicated. For steady turn, a general chart is introduced, which enables finding all turn features.

The general investigation of the unsteady turn is carried out and is applied to the following items: constant load factor turn; constant bank angle turn; constant evolution speed turn; constant angle of attack turn; brachistrochrone turn.

From author's summary

836. Bratt, J. B., and Chinneck, A., Measurements of mid-chord pitching moment derivatives at high speeds, *Aero. Res. Counc. Lond. Rep. Mem.* no. 1947, 36 pp., June 1947, published 1954.

Measurements of the pitching moment derivative coefficients $m_{\dot{\alpha}}$ and m_α for a $7\frac{1}{2}\%$ biconvex airfoil oscillating about the mid-chord axis were made in a high-speed wind tunnel by the method of decaying oscillations. The tests were made at Mach numbers of 1.275, 1.455, and 1.515 for supersonic flow, and covered a range extending from 0.4 to 0.9 at subsonic speeds. The effect of variation of frequency parameter was also investigated, and conditions giving rise to sustained or growing oscillations at subsonic speeds were examined.

Comparison with existing flat-plate theories for supersonic flow shows complete disagreement in the trend of the damping with Mach number change, the linearized theory for a flat plate giving an increasing negative value as M is reduced below 1.41, whereas experiment gives an increasing positive value. A recent theory which takes into account the shape of the profile agrees in trend with experiment, suggesting that profile is of vital importance in this field.

The results of the subsonic tests exhibit a narrow region of Mach number extending from approximately 0.87 to 0.89 within which negative damping can arise. It is thought that this effect is bound up with the formation of shock waves at the surface of the model. From authors' summary

837. Buckingham, W. R., and Lean, D., Analysis of flight measurements on the airborne path during take-off, *Aero. Res. Counc. Lond. curr. Pap.* 156, 26 pp., Oct. 1952, published 1954.

Analysis of a series of systematic take-off tests with a Meteor IV aircraft has shown that, to a good approximation, the minimum airborne path to 50 ft may be treated as an arc of a circle. With this assumption, it is a simple process to derive the mean equivalent lift coefficient used during this part of the take-off.

It has been found that the total equivalent lift coefficient used during the airborne phase decreases with increase in the ratio of the airspeed to the stalling speed in a simple manner, which is independent of the thrust/weight ratio when the shortest possible distance is required.

Using the results of a similar analysis applied to three other aircraft, an empirical rule has been developed, from which the

mean equivalent-lift-coefficient increment, and hence the minimum airborne distance to 50 ft, can be estimated simply and with reasonable accuracy.

The airborne distances thus obtained must be regarded as the minimum possible values. A factor of 1.5 may be required to allow for normal take-off techniques, particularly when the thrust/weight ratio is low. From authors' summary

838. **Schwarzbach, J. M., Boston, J. P., and Jenkins, J. E., A method of locating the airplane take-off point and determining the take-off speed,** *Aero. Engng. Rev.* **13,** 7, 42–51, July 1954.

839. **Coleman, T. L., and Fetner, M. W., An analysis of acceleration, airspeed, and gust-velocity data from a four-engine transport airplane in operations on an eastern United States route,** *NACA TN* 3483, 20 pp., Sept. 1955.

Time-history data obtained by the NACA VGH recorder from one model of a four-engine civil-transport airplane during operations on an eastern United States route are analyzed to determine the magnitude and frequency of occurrence of gusts, gust accelerations, and the associated air speeds. The results of the analysis are compared with results previously reported for two similar operations involving other types of four-engine transports. The gust-load history for the present operation is indicated to be more severe than that for the other two operations. The present airplane was operated in rough air at a higher percentage of its design speed than were the other airplanes, and this condition was primarily responsible for the more severe load history for the operation. From authors' summary

840. **Bethwaite, F. D., Some unusual aspects of turbulence,** *J. roy. aero. Soc.* **59,** 531, 220–225, Mar. 1955.

Author describes his experience with horizontal gusts of high velocity and of nearly sharp-edged character. A hypothesis is offered to explain the existence of these gusts. It is based on the premise that single or pairs of vertical vortexes, occurring under weather conditions which are conducive to the formation of tornadoes, can extend to rather high altitudes, namely, the freezing level, and lead to large horizontal gusts if traversed by aircraft.

The adequacy of present design and stress requirements with regard to the lateral gust case is questioned, as well as the safety of present piloting technique which regards lateral level as of secondary importance. H. P. Liepman, USA

841. **Bird, J. D., Some calculations of the lateral response of two airplanes to atmospheric turbulence with relation to the lateral snaking problem,** *NACA TN* 3425, 24 pp., May 1955.

Results of calculations of the lateral response to representative time histories of atmospheric turbulence for two airplanes having widely different dynamic properties indicate that, under the proper conditions, atmospheric turbulence can initiate and maintain a lateral hunting oscillation of an airplane, and that this oscillation can be fairly regular in both amplitude and frequency. This effect is more pronounced for lightly damped airplanes.
From author's summary

842. **Bryant, L. W., Halliday, A. S., and Batson, A. S., Two-dimensional control characteristics,** *Aero. Res. Counc. Lond. Rep. Mem.* no. 2730, 47 pp., 1955.

Researches on the lift, pitching moments, and hinge moments of airfoils with plain flaps have been carried out at the National Physical Laboratory at a Reynolds number of about 10^6. The results of the experiments have been presented in a generalized form, which shows promise of being applicable over a wide field. The generalized curves have been tested as far as possible from other sources, including some tests made on one of the National Physical Laboratory sections in a Royal Aircraft Establishment tunnel at Reynolds numbers up to nearly 10^7. It appears that a suggestion due to Preston that the ratio of experimental lift slope $(dC_L/d\alpha = a_1)$ to the theoretical value $(a_1)_T$, corresponding to the Joukowsky condition of flow past the trailing edge, provides a criterion giving the combined effects of Reynolds number, transition points, and airfoil shape on $dC_L/d\alpha$, and is a very useful starting point for the estimation of control characteristics. The generalized charts in this report are intended for the estimation of hinge-moment and pitching-moment derivatives from the flap/chord ratio E, after $a_1/(a_1)_T$ has been determined from a special figure. From authors' summary

843. **Goin, K. L., Theoretical analyses to determine unbalanced trailing-edge controls having minimum hinge moments due to deflection at supersonic speeds,** *NACA TN* 3471, 52 pp., Aug. 1955.

Analyses based on theoretical results of *NACA Rep.* 1041 have been made to determine the planforms of unbalanced trailing-edge flap-type controls having minimum hinge moments due to deflection and requiring minimum work to overcome the hinge moments due to deflection at supersonic speeds. Ratios of lift and rolling moment to hinge moment and ratios of lift and rolling moment to deflection work at fixed values of lift and rolling effectiveness were used as bases for the analyses.

Results of the analyses for longitudinal controls show *high-aspect-ratio untapered controls to possess maximum ratios of lift* L to hinge moment H. When low-aspect-ratio controls must be used, however, controls with triangular planforms and highly swept hinge lines are shown to have higher values of L/H than untapered controls. *Ratios of lift* to deflection work for untapered controls are, in most cases, shown to be higher than those for controls with tapered planforms.

On wings with sweptforward and *unswept trailing edges*, inversely tapered controls with triangular planforms of moderate or low aspect ratio are shown to have maximum ratios of rolling moment L' to hinge moment H. On wings with sweptback trailing edges, maximum values of L'/H are shown for either untapered or normally tapered controls. For any given control shape, the analysis illustrates the importance of using small controls with high deflections to obtain large values of L'/H.

Maximum ratios of rolling moment to deflection work on wings with sweptforward trailing edges are in most cases obtained with inversely tapered controls with triangular planforms. On wings with unswept and sweptback trailing edges, the deflection work required is near minimum for untapered controls with spans of about two thirds the wing semispan. Results indicate that large controls will, in most cases, have higher ratios of rolling moment to deflection work than smaller controls. (*From author's summary*)

Reviewer believes that the material presented will be very useful in designing control surfaces for high-speed aircraft.
A. S. Andes, USA

844. **Bland, W. M., Jr., and Sandahl, C. A., A technique utilizing rocket-propelled test vehicles for the measurement of the damping in roll of sting-mounted models and some initial results for delta and unswept tapered wings,** *NACA TN* 3314, 25 pp., May 1955.

A free-flight test technique with which the damping in roll of sting-mounted models can be measured over the high subsonic, transonic, and supersonic speed range with rocket-propelled test vehicles is described. Initial results for delta and unswept tapered wings are presented and compared with theory.
From authors' summary

845. McCarty, J. L., and Brooks, G. W., A dynamic-model study of the effect of added weights and other structural variations on the blade bending strains of an experimental two-blade jet-driven helicopter in hovering and forward flight, *NACA TN* 3367, 47 pp., May 1955.

An experimental investigation of the rotor system of a $^1/_{10}$-scale dynamic model of an experimental two-blade jet-driven helicopter to determine the effect of various changes in the design configuration on the blade-bending strains has been conducted. These changes included the addition of different amounts of concentrated weight to the blades at various radial and chordwise locations and variations in the design counterweight locations, as well as changes in the blade-pitch-control stiffness. The blade-bending strains could be materially reduced by adding concentrated weights to the blades in such a manner as to minimize resonance conditions or to induce favorable coupling of blade bending and torsion deformations.

From authors' summary

846. Imai, I., On the ascensional rate of a balloon (in Japanese), *J. meteor. Soc. Japan* (II), **32,** 7/8, 183–191, July/Aug. 1954.

847. Hitchcock, L. M., High-altitude cabin pressurization design criteria related to future transport operations, *Aero. Engng. Rev.* **14,** 8, 44–49, Aug. 1955.

Aeroelasticity (Flutter, Divergence, etc.)

848. Sisto, F., Unsteady aerodynamic reactions in airfoils in cascade, *J. aero. Sci.* **22,** 5, 297–302, May 1955.

Author presents a short summary of an important piece of research; as a matter of fact, in reviewer's opinion, the first fairly complete solution of the problem. Vibrating compressor blades have many aspects, and one important consideration is the interference effects between vibrating airfoils in cascade, which was neglected for a long time because of the great complexity. Starting from classical flutter theory and Reissner's investigations of the wall influence of a vibrating single airfoil in a wind tunnel, author formulates the integral equation for the amplitude of the vorticity distribution along the plane airfoil and its infinite wake under the later experimentally corroborated assumption of some constant phase $m \cdot \pi$ between the motions of the adjacent blades. "As compared to one isolated blade, the lift due to bending and the moment due to pitching are both changed mainly in the out-of-phase component, whereas the lift due to pitching and the moment due to bending are changed predominantly in the in-phase component. These effects increase with decreased blade spacing. It is concluded that only for exceedingly small gap/chord ratios can the effect of cascading account for any large decrease in critical flutter speeds."

Recent investigations by Wang, Lane, and Vaccaro [IAS preprint 501, 1955; and Mendelson and Caroll, *NACA TN* 3263, AMR **8**, Rev. 1761] have been undertaken in close connection with author's work and appear to confirm his numerical results both experimentally and theoretically.

In reviewer's opinion, the main trouble in actual aircraft engines is not due to the effect studied, as author rightly points out in the related extract from his summary, but it may be necessary in the future with trans- and supersonic compressors to return to author's work because of the aggravated tendencies of weaker blades in higher velocity flow, that is, larger air mass ratios $\kappa = (\pi\rho b^2/m)$ and smaller values of $k = (\omega b/v)$. Strong blade-interference effects in the form of amplitude modulation already are present in the stalled flow range, as found by the author in his doctoral thesis and in several stress-measurement investigations of actual compressor tests. It would be of great interest if the author could lay down a similar analysis for this case, which could be favorably formulated so that the results find immediate application in the aircraft engine field.

J. R. Schnittger, Sweden

849. Jordan, P. F., Some series developments in unsteady aerodynamics, *J. aero. Sci.* (Reader's Forum), **22,** 10, 722–724, Oct. 1955.

Results of aerodynamic calculations for oscillating airfoils are usually given in terms of tabulated functions; e.g., Bessel and related functions. The author of this note suggests that series representations may be more convenient for some purposes, such as machine calculations. He derives typical examples for supersonic speeds, and illustrates their utility. W. R. Sears, USA

850. Cunningham, H. J., Total lift and pitching moment on thin arrowhead wings oscillating in supersonic potential flow, *NACA TN* 3433, 43 pp., May 1955.

Formulas for total lift and moment coefficients on wide and narrow leading edge wings are constructed, by integration of section coefficients over rigid span, from linearized supersonic potential theory when trailing edge lies forward of aft Mach cone. Section coefficients are derived from first few terms (up to fifth order) in expansions of velocity potential in terms of oscillation frequency.

Present coefficients are correlated in form with the static and steady rate counterparts of rigid body stability theory and with those of an exact solution for the delta with wide leading edge.

Extensive evaluation is given of parameter variations; Mach number; leading and trailing-edge sweep. Comprehensive verification of various degrees of approximation. Increasing backward sweep destabilizes single-degree pitch. Curves for stability conditions on mass ratio and radius of gyration for two-degree pitch and heave for typical cases.

Author suggests applications in stability cases of rigid body character. P. R. Hardesty, USA

851. Hohenemser, K. H., Self-excited flapping oscillations of rotating blades at high advance ratios (in German), *ZAMM* 35, 6/7, 201–210, June/July 1955.

The theory given here restricts itself to the case of rigid blades and gives a qualitative insight into the mechanism of the subject oscillations. The influence of the main parameters and the type of the flapping motion are presented.

From author's summary by Y. C. Fung, USA

852. Woolston, D. S., Runyan, H. L., and Byrdsong, T. A., Some effects of system nonlinearities in the problem of aircraft flutter, *NACA TN* 3539, 20 pp., Oct. 1955.

The effects of nonlinear structural terms were studied on an analog computer and also by a wind-tunnel flutter model. In some cases, these effects were found to be large and destabilizing and, as expected, dependent on the magnitude of the external excitation. B. Smilg, USA

853. Bosschaart, A. C. A., The influence of the chord, span, and gear ratios on binary aileron-spring tab flutter, *Nat. LuchtLab. Amsterdam Rap.* no. F. 166, 14 pp. + 44 figs., 1955.

Paper is the product of further theoretical study at NLL on the flutter of aileron-spring tab systems. Earlier work of van de Vooren and Hofsommer [title source, *Rap.* F.52, Sept. 1949; AMR **3,** Rev. 2756] has been expanded into a more complete survey from which have been derived general criteria for flutter-

free aileron-spring tab systems of various chord, span, and gear ratios. This survey has been made for stick-fixed oscillations of aerodynamically unbalanced aileron and tab. As in the previous investigation (cited above), the aileron and tab have each been assumed structurally rigid within themselves and have been fitted to a fixed rectangular wing. Compressibility effects and structural damping have not been considered. Results are presented in the form of diagrams showing which combinations of aileron and tab moments of inertia and tab static moment are stable at any airspeed. The proposed stability criteria are derived from these results and compared with the results of those derived by Collar and Sharpe [*RAE Rep.* nos. SME 3346, 1944; and SME 3378, 1946] and by Wittmeyer and Templeton [*RAE Rep.* no. Structures 57, 1950].

From author's summary by J. L. Sewall, USA

854. **McNeill, W. E., and Cooper, G. E., A comparison of the measured and predicted lateral oscillatory characteristics of a 35-degree sweptwing fighter airplane,** *NACA TN* 3521, 22 pp., Aug. 1955.

Propellers, Fans, Turbines, Pumps, etc.

(See Revs. 649, 664, 686, 687, 808, 866, 876, 941, 945, 950, 953)

Flow and Flight Test Techniques

(See also Revs. 776, 799, 809, 831, 842, 852, 896, 897, 908, 932, 947)

855. **Schotland, R. M., The measurement of wind velocity by sonic means,** *J. Meteor.* **12,** 4, 386–390, Aug. 1955.

The theory of the measurement of wind velocity by sonic means is presented. Two methods are discussed for determination of the rectangular components of the wind velocity vector by use of the sonic principle. The response characteristics and application of sonic anemometers are discussed. A sonic anemometer for the measurement of the horizontal component wind vector is described, which may be primarily used as a research instrument for use in turbulence studies.

From author's summary by S. Apáti, Hungary

856. **Richardson, F. M., Ferrell, J. K., Lamonds, H. A., and Beatty, K. O., Jr., How radiotracers are used in measuring fluid-velocity profiles,** *Nucleonics* **13,** 7, 21–23, July 1955.

New tracer technique determines velocities within 0.002 in. of the wall—much closer than other methods—in velocity-gradient studies. Instruments used record 10-fold range in counting rate without need for changing scaling factor or chart speed.

From authors' summary

857. **deHaller, P., Flow measurement with Venturi tubes at high Reynolds numbers,** *Sulzer tech. Rev.* **37,** 1, 46–53, 1955.

858. **Tollenaar, D., and Bisschop, M. C., The bar viscometer,** *J. coll. Sci.* **10,** 2, 151–155, Apr. 1955.

859. **Banwell, C. J., Flow-sampling and discharge measurement in geothermal bores,** ASME Advance paper no. 55—A-97, 21 pp. + 16 figs., 1955.

860. **Ludwieg, H., The tube wind tunnel** (in Germzn), *Z. Flugwiss* **3,** 7, 206–216, July 1955.

Author proposes new type of wind tunnel intended to produce given Mach and Reynolds numbers at minimum cost. New wind tunnel is essentially intermittent-flow blow-down tunnel with air supply stored in a long tubular vessel (300 m long in example). Tube is connected directly to De Laval nozzle or to subsonic diffuser followed by nozzle, thence to parallel test section and a supersonic diffuser, ending in quick-opening valve. Discharge may be to atmosphere or to vacuum tank. Through travel of expansion wave in supply tube, flow at constant stagnation pressure and temperature is maintained for one or two seconds. Influence of boundary layer and means for alleviating its effects are discussed. Possible experiments and measuring techniques are described. H. L. Dryden, USA

861. **Knowler, A. E., and Holder, D. W., The efficiency of high-speed tunnels of the induction type,** *Aero. Res. Counc. Lond. Rep. Mem.* no. 2448, 59 pp., 1954.

Collection of results from twelve reports which describe development work on induction wind tunnels. Mach number range was from 0.4 to 1.4. Results show relatively high efficiency at subsonic speeds but rapid decrease with increasing supersonic speed. Data show effects of changing many pertinent variables.

Reviewer suggests report by Spiegel, Hofstetter, and Kuehn [*NACA RM* A53I01, "Applications of auxiliary air injectors to supersonic wind tunnels," 1952] as an important additional reference. W. F. Davis, USA

862. **Enkenhus, K. R., and Tucker, N. B., Design and testing of axisymmetric flow nozzle for Mach number seven,** *Univ. Toronto Inst. Aerophys. Rep. tech. note* no. 4, 27 pp. + lxxxiv, Jan. 1955.

A hypersonic nozzle of circular cross section was first designed by potential theory. The stream tube for a square outlet was calculated backward from the outlet to determine a nozzle of square cross section. A mahogany nozzle was built and tested with unheated air at a stagnation pressure of one atmosphere. Nozzle surveys show a region of low pitot pressure near the wall followed by a rapid transition to a higher pressure in the central portion of the nozzle. The flow appears separated with a circular jet in the center. These results are qualitatively similar to low Reynolds number data of Wegener [*J. aero. Sci.* Preprint no. 329, 1951] with the following qualifications: Wegener results are for no condensation and for nozzle expansion in only one plane. Also, Wegener obtained a pitot pressure defect near the center of the nozzle, while present authors exhibit no data for this position. J. Nielsen, USA

863. **Thompson, J. R., Bray, R. S., and Cooper, G. E., Flight calibration of four airspeed systems on a swept-wing airplane by Mach numbers up to 1.04 by the NACA radar-phototheodolite method,** *NACA TN* 3526, 41 pp., Nov. 1955.

The calibrations of four airspeed systems installed in a North American F-86A airplane have been determined in flight at Mach numbers up to 1.04 by the NACA radar-phototheodolite method. The variation of the static-pressure error per unit indicated impact pressure is presented for three systems typical of those currently in use in flight research—a nose boom and two different wing-tip booms—and for the standard service system installed in the airplane. A limited amount of information on the effect of airplane normal-force coefficient as the static-pressure error is included. The results are compared with available theory and with results from wind-tunnel tests of the airspeed heads alone.

Of the systems investigated, a nose-boom installation was found to be most suitable for research use at transonic and low supersonic speeds because it provided the greatest sensitivity of the indicated Mach number to a unit change in true Mach number at very high subsonic speeds, and because it was least sensitive to changes in airplane normal-force coefficient. The static-

pressure error of the nose-boom system was small and constant above a Mach number of 1.03 after passage of the fuselage bow shock wave over the airspeed head.

From authors' summary

864. Thomas, D. F., Jr., Low-speed static lateral and rolling stability characteristics of a series of configurations composed of intersecting triangular plan-form surfaces, *NACA TN* 3532, 29 pp., Oct. 1955.

The static lateral and rolling stability derivatives of a series of cruciform, inverted T-, V-, and Y-configurations composed of low-aspect-ratio triangular surfaces have been obtained at low speed in the 6-ft-diam rolling-flow test section of the Langley stability tunnel. These derivatives are presented as functions of the geometry of the models and, for two configurations (a planar wing and an inverted T), as functions of angle of attack. Where possible, comparisons have been made to indicate the extent of agreement between experiment and existing theory. In general, the sideslip derivatives showed better agreement between theory and experiment than the rolling derivatives.

From author's summary

865. Spiedel, L., Measurement on two laminar airfoil sections for gliders (in German), *Z. Flugwiss.* **3,** 10, 353–359, Oct. 1955.

The laminar airfoil section FX 1057–816 theoretically developed for a glider by F. X. Wortmann, and the corresponding American laminar airfoil section NACA $65_{(215)}$–714, have been measured over the range of Reynolds numbers from $0.6 \cdot 10^6$ to $1.6 \cdot 10^6$. The profile drag was obtained from wake traverse measurements; the normal force and the moment coefficients from integration of the pressure distribution. A comparison of the polar diagrams gives the same minimum drag but a higher design c_a and a wider trough of low drag for the FX than for the NACA airfoil section. From author's summary

866. Hammack, J. B., Windler, M. L., and Scheithauer, E. F., Flight investigation of the surface-pressure distribution and the flow field around a conical and two spherical nonrotating full-scale propeller spinners, *NACA TN* 3535, 36 pp., Sept. 1955.

A flight investigation has been made on three nonrotating full-scale propeller spinners at free-stream Mach numbers of 0.70 to 0.96 to determine the surface-pressure distribution and the magnitude and radial extent of the influence of each spinner on the local flow field. One spinner was a conventional conical spinner. The other two spinners had conical forebodies and spherical midsections that provided a mechanically simple means of affording an aerodynamically clean blade seal.

The departure from stream conditions in the propeller plane of the conical spinner and the 60° spherical spinner extended beyond the 1.3-spinner-radius station; whereas, with the 45° spherical spinner, stream conditions were reached at the 1.3-spinner-radius station. High peak negative pressure coefficients occurred over the spherical section of both spherical spinners between the cone-sphere juncture and the blade centerline. The local surface Mach numbers in this region are approximately 0.4 higher than that at free-stream Mach number of 0.95. The effect of yaw on both the surface-pressure distribution and the local flow field was investigated over the top and bottom surfaces of the 45° spherical spinner and was found to be small for the test conditions.

Unfavorable flow separation occurred over the rear portion of the spherical spinners. This flow separation caused mild buffeting of the airplane above a free-stream Mach number of 0.72 with the 60° spherical spinner and above 0.83 with the 45° spherical spinner. From authors' summary

867. Jaquet, B. M., and Williams, J. L., Wind-tunnel investigation at low speed of effect of size and position of closed air ducts on static longitudinal and static lateral stability characteristics of unswept-midwing models having wings of aspect ratio 2, 4, and 6, *NACA TN* 3481, 45 pp., Sept. 1955.

An investigation was made at a Mach number of 0.13 in the Langley stability tunnel in order to determine the effects of closed wing-root air ducts (horizontal) on the static longitudinal and static lateral stability characteristics of unswept-midwing models having wings of aspect ratio 2, 4, and 6. In addition, the effects of top and bottom fuselage ducts (vertical) on the static longitudinal and static lateral stability characteristics of model configurations employing the unswept wing of aspect ratio 2 were determined.

The results of the investigation have indicated that, in the low angle-of-attack range, the addition of and increase in size of the horizontal ducts on model configurations employing an unswept wing of aspect ratio 2 resulted in a large forward movement of the aerodynamic center regardless of the vertical location of the horizontal tail. When the aspect ratio of the wing was increased from 2 to 6, this effect became more pronounced. In contrast to this effect of the horizontal ducts, the addition of and increase in size of vertical ducts on model configurations employing the wing of aspect ratio 2 produced a slight rearward movement of the aerodynamic center.

Regardless of the aspect ratio of the wing, the addition of and increase in size of the horizontal ducts caused an increase in directional stability for complete models or a decrease in instability for tail-off configurations at low and moderate angles of attack. The addition of and increase in size of vertical ducts on the models with the wing of aspect ratio 2, however, resulted in large decreases in directional stability which were about constant for the angle-of-attack range investigated.

From authors' summary

868. Jones, R., Miles, C. J. W., and Pusey, P. S., Experiments in the compressed air tunnel on swept-back wings including two delta wings, *Aero. Res. Counc. Lond. Rep. Mem.* no. 2871, 26 pp., Mar. 1948, published 1954.

The investigation was undertaken to provide data relating to C_L, C_D, and C_m at high values of Reynolds number on wings of triangular planform, delta wings. C_L, C_D, and C_m were measured over a range of R from 0.5×10^6 to 8 or 9×10^6 and a range of incidence from zero to above the stall.

The models tested comprised:

(*a*) Delta 1, whose planform was a right-angled isosceles triangle of span approximately 4 ft and aspect ratio 3.87. The span was twice shortened by removing sections from the tips, giving aspect ratios of 3.04 and 2.38. The model of aspect ratio 3.04 was also tested with a straight flap and with a body. This model was also tested in a modified form with the leading-edge radius increased from 0.0069*c* to 0.018*c* by decreasing the local chord *c* by 1.5%.

(*b*) Delta 2 of equilateral triangular planform, side 3 ft and aspect ratio 2.31. This model was also tested with a flap (60 deg) and with a body, the former being tried in two positions: (1) near the trailing edge, and (2) 10 in. forward of the trailing edge.

(*c*) Delta 3, a conventional sweptback arrowhead wing of aspect ratio 3.07. No tests with body were carried out on this model, but a straight flap perpendicular to the centerline and a "V" flap with arms parallel to the trailing edge were tried. The section of the three original models was 10% thick, with the maximum thickness at 0.35 of the chord from the leading edge.

From authors' summary

869. **Hall, I. M., Experiments on supersonic flow over flat-nosed circular cylinders at yaw—II: Pressure measurements on a cylinder at 10° yaw,** *Phil. Mag.* (7) **46,** 372, 53–60, Jan. 1955.

Author measures pressure distribution over nose of yawed flat-faced cylinder at Mach number of about 2.0. Close to the nose a large cross force was found to exist which was attributed to the asymmetry of boundary-layer separation. Further downstream the pressure distributions resemble those obtained about an infinite cylinder started impulsively from rest. [For part I see AMR **7,** Rev. 2587.] G. Goodwin, USA

870. **Oshima, K., Shock tube study of the flow past circular-arc airfoils using a Mach-Zehnder interferometer,** *J. phys. Soc. Japan* **10,** 7, 571–577, July 1955.

The shock tube is used to measure the flow over two-dimensional circular-arc airfoils in the transonic speed range. The measurements are carried out by means of an interferometer and used to check the transonic similarity laws. Paper conveys the impression that results obtained in a shock tube are free of blockage interference at near sonic speeds. Reviewer does not agree with this point of view. While no chocking can be observed in a shock tube if the nominal Mach number is taken to be the one behind the shock wave, the same wall interference will nevertheless be present as in a conventional tunnel employing a flexible nozzle to permit starting in the upper transonic regime.

P. F. Maeder, USA

871. **Taylor, J., Gusts and their measurement,** *J. roy. aero. Soc.* **58,** 528, 826–832, Dec. 1954.

Thermodynamics

(See also Revs. 684, 783, 793, 799, 808, 810, 811, 858, 886, 895, 915, 931, 942, 950)

872. **Norris, E. B., Therkelsen, E., and Trent, C. E., Applied thermodynamics,** 3rd ed., New York, McGraw-Hill Book Co., Inc., 1955, xviii + 490 pp. $7.50.

This is a revision of the 1939 edition of "Heat power" by Norris and Therkelsen, the revision primarily involving the addition of (fairly brief) material on compressors and gas turbines. The general emphasis of the book is on practical matters, and the descriptive sections and photographs are indeed excellent. The reviewer's sole criticism is that the treatment of basic thermodynamics is scattered and perhaps somewhat shallow. For example, the definition of entropy is accomplished as follows: "Now we may devise a new thermal quantity by dividing this heat change *dq* by the absolute temperature at which the heat change takes place, ..., and we may call this new quantity *entropy change*. It should be noted that this quantity is a purely mathematical expression of the same nature as the moment of inertia of a beam section, and, so far as this discussion is concerned, need not be considered to have a real existence any more than moment of inertia has."

A summary of the contents follows: General survey of heat power, particularly public utilities. Internal-combustion engine: qualitative explanation and construction details. Diesel engines. Practical problems of combustion, lubrication, ignition. Survey of fuels. Chemistry of combustion (but no treatment of flame speeds or detonation waves). Measurement of work and power. Indicator diagrams. Measurement of heat. Gas laws. Flow processes. Area under polytropic curve in PV diagram. Entropy, with exemplary calculations. Heat in a TS diagram. Heat engine cycles. Comparison of theoretical efficiency with actual performance. Steam plants. Enthalpy and Mollier diagram. Performance and improvement of efficiency. Steam turbine. Impulse calculations. Condensers and boilers. Furnaces and fuels. Compressor cycles. Rotary compressors. Refrigerators. Thermodynamic tables.

C. E. Pearson, USA

873. **Wójcicki, S., Thermodynamic problems of ramjet engines** (in Polish), *Techn. Lotn.* **9,** 3, 66–68, May/June 1954.

874. **Köhler, J. W. L., and Jonkers, C. O., Fundamentals of the gas refrigerating machine,** *Philips tech. Rev.* **16,** 3, 69–104, Sept. 1954.

Philips gas refrigerating machine was first dealt with in title source **8,** p. 136, May 1946, where H. Rinia and F. K. du Pré revealed the big development work started during World War II to develop a good hot-gas engine according to modern technics. Hot-air engines were originally developed by Stirling Brothers (1817) and John Ericsson (1833). The theory was described in Rinia's paper, 1946. (Reviewer finds a similar theory already dealt with in Zeuner, "Technische Thermodynamik," Band 1, Leipzig 1887, pp. 347–357.)

Rinia and du Pré stated that the machine could easily be used as a gas refrigerating machine. The machine could reach a temperature of 80 K in one stage. Köhler and Jonkers describe the principle of the hot-gas engine applied to a gas refrigerating machine (see also Zeuner, pp. 374–377). A gas quantity flows to an expansion compartment where it expands under absorption of heat and is then compressed in another compartment with higher temperature, whereupon the gas flows back to the expansion room. During the gas transfer from one compartment to the other the gas volume is almost constant in the Stirling machine, whereas in the Ericsson machine the pressure is constant Stirling (1832), as well as Ericsson (1833), introduced a regenerator between the two compartments, and it is easily proved that with 100% efficiency of the regenerator the same coefficient of performance is obtained as with a Carnot process.

Authors state that the use of the Stirling machine as a refrigerating machine was dealt with already in 1834 by John Herschel, and a patent for such a machine was granted to Alexander Kirk in 1862. They state that if, in a hot-gas engine, the heat supply is discontinued to the compartment which is normally supplied with heat, and an electric motor drives the engine, said compartment will continue to absorb heat and will therefore cool rapidly until its temperature drops below the temperature of the machine section where heat is removed, for instance, by means of cooling water. The difference between a hot-gas engine and a cold-gas machine is thus, in principle, almost nil. A number of machines of this type are at present in use in Philips factory in Eindhoven for generation of refrigeration at about −200 C.

M. Bäckström, Sweden

875. **Köhler, J. W. L., and Jonkers, C. O., Construction of a gas refrigerating machine,** *Philips tech. Rev.* **16,** 4, 105–140, Oct. 1954.

Authors' previous article (see preceding review) continues here and deals first with various losses. A loss of 1% in the regenerator reduces the refrigerating capacity by 21% in a liquid-air machine. If 20 K could be reached, a regeneration loss of 1% corresponds to 98% in refrigerating capacity. The machine designed by Philips laboratories is described by drawings showing that the machine is quite similar to the Stirling and Lehmann machines with regenerator. A machine of very simple construction is described, but reviewer thinks that, although based on principles known 100 years ago, the working principle is not easy to understand even for a man skilled in the art. The small ver-

tical machine has a cylindrical top cold enough for liquid air to condense here. The machine works with a single stage. 5.7 kw gives 5.8 kg liquid air per hour and this air drips down directly from the cold cylinder head. Test results show that, at −130 C, the coefficient of performance obtained is 40% of that at the Carnot process.

Reviewer believes that the machine described is one of the most ingenious refrigerating machines that have been developed in the past decade, even if such a machine was invented more than 100 years ago. M. Bäckström, Sweden

876. Köhler, J. W. L., and Jonkers, C. O., Gas refrigerating machines (in Dutch), *Ingenieur* **66,** 51, 0.103–0.110, Dec. 1954.

Gas refrigerating machines are refrigerators based on the Stirling cycle. This cycle is performed with a nearly perfect gas compressed at ambient and expanded at a low temperature. Machines of this type are particularly suitable to produce cold between −80 C and −200 C, which range is reached in a single stage. The principles of the process are discussed, as well as the deviations occurring with the realization. If working at −200 C, these machines may condense atmospheric air. An air liquefier of this type is described; it has an output of about 6 liters/hr. As there is no need of compressing the air to be liquefied, conspicuous simplifications are possible. Finally, a survey is given of the possible applications of this reborn refrigeration process.
From authors' summary

877. Hicks, T., Refrigeration, *Power* **99,** 10, 75–94, Oct. 1955.

878. Dzung, L. S., Thermostatic change of state of dry and wet steam (in German), *ZAMP* **6,** 3, 207–223, 1955.

General thermodynamic relations for a simple system (no action, capillarity, etc., and no chemical reaction) are derived in terms of the quantities

$$k \equiv (\partial \ln p/\partial \ln v)_s \qquad \lambda \equiv k - 1/k$$

$$m \equiv (\partial \ln p)/(\partial \ln v)_h \qquad \mu \equiv m - 1/m$$

$$\tau \equiv pv/R \qquad z \equiv pv/RT = \tau/T \qquad \phi \equiv dh/vdp$$

These are applied to certain special cases:

Ideal gas: τ/T = constant.

Ideal vapor: $m = 1$ (or $\mu = \infty$), for which it is shown that h is a function of τ only and z is a function of entropy only.

Semi-ideal vapor: $\lambda = \lambda(\tau)$ and $\mu = \mu(s)$, for which it is shown that μ is constant. Relations are given for constant s, h, p, and ϕ.

Wet vapor: ideal gas mixed with constant-volume liquid. Variation of k and m with x is shown. For $x > 0.6$, k and m are nearly constant and mixture conforms approximately to relations of semi-ideal vapor.

Supersaturation: Expression derived for entropy increase in condensation from a nonequilibrium mixture in pressure-temperature equilibrium to a mixture in thermodynamic equilibrium without change in p or h.

The relations derived should be of use in design of multistage machinery using wet or dry vapors. J. Keenan, USA

879. Price, D., Thermodynamic functions of carbon dioxide, enthalpy, entropy, and isobaric heat capacity at 100 to 1000 C and 50 to 1400 bars, *Indust. Engng. Chem.* **47,** 8, 1649–1652, Aug. 1955.

At the present time, tables of thermodynamic functions based on pressure-volume-temperature (P-V-T) measurements of carbon dioxide gas are available in the ranges 25 to 150 C, 1 to 3000 atm, and 0 to 600 C, 1 to 50 atm. Recently, P-V-T data have been determined by Kennedy at higher simultaneous temperatures and pressures. These data have been used in the present work to compute thermodynamic functions in the range 100 to 1000 C, and 50 to 1400 bars. From author's summary

880. Eakin, B. E., Ellington, R. T., and Gami, D. C., Physical-chemical properties of ethane-nitrogen mixtures, *Inst. Gas Tech. Res. Bull.* 26, 40 pp., July 1955.

This bulletin presents data on liquid-vapor phase equilibria, gas phase pressure-volume-temperature relationships, and saturated liquid and vapor densities for the ethane-nitrogen system, obtained by detailed experimental investigation with specific mixtures of ethane and nitrogen.

The liquid-vapor phase equilibrium data were determined by study of the dew and bubble-point pressure-temperature relations of ten mixtures of ethane and nitrogen. During experimental runs the mixture was confined in a heavy-walled glass cell immersed in a cryostat, and stirred by a magnetically raised and lowered steel ball. Dew and bubble points were determined by direct visual observation, and the critical region of the mixture was carefully noted if critical conditions were within the operating limits of the equipment. The range covered was from 50 to 1950 psia, and −297 to +90 F, the critical temperature of ethane. It was not possible to determine the highest pressure at which liquid and vapor can coexist, due to equipment limitations.

Pressure-temperature, temperature-composition, and pressure-composition diagrams were prepared from the dew and bubble point data. Equilibrium vaporization ratios were determined from these diagrams, and are presented in graphical and tabular form. It is estimated that the phase equilibrium data are reliable to ±2 psi or ±0.2 F, whichever is greater.

The gas-phase pressure-volume-temperature data were determined by a study of the pressure-temperature relations of five mixtures of ethane and nitrogen at constant density. Seven or eight constant density lines (isometrics) were investigated for each mixture. The data, covering the range from the critical region to +110 F and pressures to 4000 psia, are presented in plots of compressibility factor versus pressure, with lines of constant temperature. Compressibility factors calculated from the data are believed accurate to ±0.3%.

The experimental compressibility factors have been compared with values calculated with the Benedict-Webb-Rubin equation of state, the constants for the mixtures being obtained by combining the constants for ethane and nitrogen. An empirical combination rule for the A_0 term was used to adjust the method of combining constants and improve the agreement with the experimental data. A comparison of the experimental pressure-volume-temperature data with the data of Reamer, et al., show good agreement, with a maximum deviation of about 1.2%.
From authors' summary

881. Rothman, A. J., and Bromley, L. A., High temperature thermal conductivity of gases; measurements on nitrogen, carbon dioxide, argon, and nitrogen–carbon dioxide mixtures at temperatures up to 775 C, *Indust. Engng. Chem.* **47,** 5, 899–906, May 1955.

A silver cell for the measurement of the thermal conductivity of gases, together with necessary auxiliary equipment, has been built and used for the measurement of the thermal conductivity of nitrogen, carbon dioxide, argon, and nitrogen–carbon dioxide mixtures up to about 775 C. Suggested values of the thermal conductivity of nitrogen and carbon dioxide based on the experimental data are given in Table III. Graphs are presented for nitrogen, carbon dioxide, argon, and nitrogen–carbon dioxide mix-

tures. The values shown on the graphs at the highest temperatures are probably a few per cent high.

From authors' summary

882. **Dougherty, E. L., Jr., and Drickamer, H. G., A theory of thermal diffusion in liquids,** *J. chem. Phys.* **23,** 2, 295–309, Feb. 1955.

Thermal diffusion phenomena for binary systems are expressed alternately in terms of the thermal diffusion ratio α or the net heats of transport Q_1^* and Q_2^* for the two components where $\alpha = (Q_2^* - Q_1^*)/[x_1(\partial\mu_1/\partial x_1)]$. By considering a simple model this net heat of transport can be shown to equal the net amount of energy which must be absorbed per mole of the component diffusing out in order to maintain constancy of temperature and pressure. Accordingly, this may be related to the interaction energies involved in replacing one component by the other. These interaction energies are expressible in terms of the excess energy of mixing and the cohesive energy, i.e., essentially the energy of vaporization, together with factors dependent on the relative volumetric packing of the two molecular species and on the effective activation energy for transport phenomena. The determination of α is thereby reduced to evaluating more fundamental parameters. Experimental data are examined by which the relation between these parameters and molecular structure is demonstrated. N. A. Hall, USA

883. **Fraser, M. H., A useful correlation of some physical properties of liquids (Research Correspondence),** *Research, Lond.* **8,** 11, S57–S58, Nov. 1955.

During a survey of data on vapor pressure and other liquid properties at the same temperature, a correlation was found of the form: $\eta^2 p^\alpha/\gamma = \beta$, where η is viscosity (poise), p vapor pressure (mm Hg), γ surface tension (dyne/cm); α and β are dimensional constants. From author's summary

884. **Ramsay, J. A., and Brown, R. H. J., Simplified apparatus and procedure for freezing-point determinations upon small volumes of fluid,** *J. sci. Instrum.* **32,** 10, 372–375, Oct. 1955.

For purposes of determining the freezing point of small volumes of aqueous solutions the difficulties of undercooling are avoided by first freezing the sample and then determining the thawing-point. Apparatus and procedure specially designed for simplicity of construction and operation are described. The method works best with volumes of the order of 10^{-3} to 10^{-4} mm³ and its accuracy in terms of standard deviation is ±0.003 C for freezing-point depressions of the order of 1 to 2 C.

From authors' summary

885. **Evans, W. H., Jacobson, R., Munson, T. R., and Wagman, D. D., Thermodynamic properties of the alkali metals,** *J. Res. nat. Bur. Stands.* **55,** 2, 83–96, Aug. 1955.

The available data on thermodynamic properties of lithium, sodium, potassium, rubidium, and cesium are reviewed and tabulated. R. Betchov, USA

Heat and Mass Transfer

(See also Revs. 683, 772, 784, 787, 799, 820, 874, 875, 876, 879, 881, 885, 922, 936, 940)

886. **Grigull, U., Gröber, and Erk, Basic laws of heat transmission [Grundgesetze der Wärmeubertragung],** Berlin, Springer-Verlag, 1955, viii + 428 pp. DM 37.50.

This text was originally written by H. Gröber in 1921 essentially as a treatise on heat conduction. The presentation of the mathematical theory of conduction was so excellent that the book became a classical text in the German literature dealing with this field. In the second edition edited in 1933 by S. Erk, a discussion of the basic physical laws and the mathematical relations describing convective heat transfer as well as a short treatise on heat radiation were added. In the present edition written by U. Grigull, the first chapter on heat conduction remains again essentially unchanged with the exception of a few additions. Diagrams indicating the temperature history in a cooled slab, cylinder, and sphere are replaced by newer ones calculated by Bachmann. A discussion of the relaxation method and a chapter on rarified gases are added.

That part of the book dealing with convection, however, was completely rewritten and considerably enlarged. It now comprises more than half of the book. This section gives a very thorough and up-to-date account of our knowledge in this field. The main emphasis was put on presentation and coordination of available information. In order to save space, the derivation of Navier-Stokes equations and of the energy equation has been eliminated, a fact which may be deplored by some readers who would like to see these derivations included in a text on basic laws of heat transfer. The wealth of material included in the convection chapter, however, well compensates for this loss. The similarity laws as the basis of convective heat-transfer relations, heat transfer in forced laminar and turbulent flow, heat transfer on bodies in oblique flow and at high velocities, in free convection, condensation, and evaporation, and mass transfer especially in its analogy to the transfer of heat are discussed, and each chapter ends with a tabulation of recommended relations. The chapters on condensation and evaporation should be pointed out as especially well written. The first one of these chapters also contains new work of the author in this field.

The third part of the book dealing with heat radiation has been somewhat extended by including new information on over-all and monochromatic emissivities and absorptivities of solids, liquids, and gases. The properties listed in the appendix of the book have been brought up to date.

A few minor errors or omissions occurring in the test are listed for possible correction in a new edition. The equation describing the unsteady temperature field connected with heat conduction in a body with temperature-dependent properties is listed on page 130. The heat conduction term in this equation should properly read

$$(1/c\gamma)\nabla(k\nabla\vartheta)$$

It would also be of advantage in this connection to point out, specifically in the presentation of the equations describing convective heat flow, where they are restricted to constant property values (as, for instance, Eq. 18 on p. 151). There is presently a considerable discussion under way on whether the viscous terms in Eq. 10e on p. 145 are the proper ones for a compressible fluid. It would have been advisable to include in the discussion on mass transfer the considerable number of investigations which have been made, especially in the United States, on transpiration cooling.

In summary, the book is a very valuable addition to the literature in the field of heat transfer and will certainly be widely used not only by students who want to acquaint themselves with this area, but also by research workers as a reference text.

E. R. G. Eckert, USA

887. **Taylor, C. L., Heat-transfer applications in the human body,** *Mech. Engng., N.Y.* **77,** 6, 511–513, June 1955.

A brief review of the factors influencing the thermal balance of the human body with its surroundings is presented, with some historical notes on development of the currently availabie information. An electric analog model is described in principle with

which it is hoped the complex heat and mass-flow problems of the transient human-environment relationships can be solved.

F. C. Hooper, Canada

888. Gordov, A. N., Temperature of an infinite cylinder in a flow with pulsating velocity and temperature (in Russian), *Prikl. Mat. Mekh.* **19**, 240–243, 1955.

Paper deals with the problem of response of a thin cylindrical probe placed in a pulsating stream. The difference between the present and standard solution lies in the fact that the coefficient of heat transfer, α, has not been assumed constant. Postulating that the pulsating temperature $t(\tau) = t_m + A_1 \cos \omega\tau$ (τ = time, t_m = mean temperature) is accompanied by a synchronously pulsating velocity, author assumes that the Biot (i.e., Nusselt) number $hR = (\alpha/\lambda)R$ varies in the same manner

$$hR = h_0R + A_2R \cos \omega\tau$$

(λ thermal conductivity of fluid, R outer radius of cylinder). No discussion of the physical basis of this, otherwise plausible, assumption is given.

Author then proceeds to solve the heat-conduction equation in cylindrical, polar coordinates (assuming, again without discussion of the physical implications, that $\partial/\partial\varphi = 0$) for an infinite cylinder, where the body temperature is $\theta(r, \tau)$. Having set up the appropriate initial and boundary conditions, author solves the problem by the use of the Laplace transform method. The final solution (rather complicated) is obtained in the form of an infinite series, whose first five terms have been given explicitly for the solution for the mean temperature $\bar{\theta}(\tau)$ (the mode of averaging over the radius r is not indicated).

The following conclusions are drawn: (1) The mean cylinder temperature is shifted with respect to the mean fluid temperature t_m by a constant $M_1 = \frac{1}{2} A_1A_2R/(h_0R + A_2R$ ber $HR)$, $H = (\omega/a)^{\frac{1}{2}}$, which is a measure of the inherent error in response. The influence of a phase angle φ_0 between the coefficient of heat transfer and temperature is also given in the paper. The shift $M_1 = 0$ for $\varphi_0 = \pm \frac{1}{2} \pi$. (2) The cylinder temperature is the sum of two harmonics so that it is not symmetric even if the fluid temperature is sinusoidal. Thus an asymmetrical response is not necessarily the result of a nonharmonic forcing function.

J. Kestin, USA

889. Giulianini, A., On the heat transfer in finned tubes (in Italian), *Termotecnica* **8**, 6, 347–351, June 1954.

The problem of heat transfer through finned tubes, ordinarily treated by means of experimental proofs, is here treated analytically, in stationary conditions, by means of Bessel functions. Author gives numerical applications of the results and compares these with the experimental data. Obviously, the differences decrease with increasing of dimensions of the diameters of the tubes and of the fins, and with decreasing of the number of the fins.

A. Pignedoli, Italy

890. Barbaro, D., Heat penetration in electric conductors with high frequency (in Italian), *Termotecnica* **8**, 6, 352–354, June 1954.

Author reduces the problem analytically to the consideration of heat penetration in a cylinder (very long with respect to the dimensions of the base) when the cylinder is immersed in an electromagnetic field extending (with plane waves) along the axis, producing heat exclusively at the base. Author gives a practical example, graphically treated.

A. Pignedoli, Italy

891. Baratta, M. A., On a problem of heat conduction (in Italian), *Riv. Mat. Univ. Parma* **5**, 1/2/3, 197–207, 1954.

Two coaxial cylindrical bodies, S_1, S_2, are separated by a cylindrical boundary carrying sources of heat. Author determines temperatures of S_1 and S_2 as functions of r and θ if initial temperature distributions in S_1 and S_2 are given functions not only of the distance from axis, but of both polar coordinates r and θ in a normal section, and if temperature of sources is a given function of θ and time.

Z. Horák, Czechoslovakia

892. Greebler, P., Thermal properties and applications of high-temperature aircraft insulation, *Jet Propulsion* **24**, 6, 374–378, Nov./Dec. 1954.

This paper describes the characteristics of high-temperature aircraft insulation and it is written for the general reader who is interested in the applications. This is to say, it is not a paper on new research written primarily for other researchers.

Author computes the relative contributions due to fiber conduction, air conduction, and radiation for a high-temperature felt at various temperatures. The influence on thermal conductivity of other factors such as fiber diameter, density, and foil emissivity is discussed. Several representative problems are solved to illustrate the behavior of the insulation under both steady-state and transient conditions.

D. C. Hamilton, USA

893. Deissler, R. G., Turbulent heat transfer and friction in the entrance regions of smooth passages, *Trans. ASME* **77**, 8, 1221–1233, Nov. 1955.

The effect of various factors on the turbulent heat transfer and friction in the entrance regions of smooth passages is investigated analytically. The influence of Reynolds number, Prandtl number, initial velocity distribution, wall-boundary condition, passage shape, and of variable fluid properties is predicted. Integral heat-transfer and momentum equations are used for calculating the thicknesses of the thermal and flow boundary layers. The results indicate that approximately fully developed heat transfer and friction are, in general, attained in an entrance length less than 10 diameters. Substantial agreement between analysis and experiment was obtained for heat transfer to air in the entrance regions of tubes and parallel plates.

From author's summary by R. L. Young, USA

894. Bartz, D. R., An approximate solution of compressible turbulent boundary-layer development and convective heat transfer in convergent-divergent nozzles, *Trans. ASME* **77**, 8, 1235–1245, Nov. 1955.

The boundary-layer integral momentum and integral energy equations for thin boundary layers in axisymmetric nozzles are used together with the assumptions (1) that both the profile of the velocity across the boundary layer and the profile of the difference between stagnation temperature and wall temperature are given by the "1/7 power-law"; (2) that the local skin-friction coefficient is the same function of the local boundary-layer thickness as on a flat plate, the flat plate friction formula used includes the Mach number and wall temperature ratio as parameters; and (3) that the heat-transfer coefficient is related to the friction coefficient by a formula that includes the Prandtl number and the ratio of the thermal to the velocity boundary-layer thickness. To simplify the integration of the integral momentum and integral energy equations (two simultaneous ordinary differential equations), the recommended procedure is to first integrate the momentum equation approximately and then to integrate the energy equation. The integration gives the distribution of the velocity and temperature boundary-layer thickness along the nozzle.

The computed boundary-layer thicknesses and heat-transfer coefficients for a nozzle are shown for different boundary-layer thicknesses at the nozzle entrance. The computed heat-transfer coefficients are also compared with some available experimental data and the agreement is good. N. Tetervin, USA

895. Synge, J. L., The motion of a viscous fluid conducting heat, *Quart. appl. Math.* **13,** 3, 271–278, Oct. 1955.

Author attacks "entrophoby"—defined as the abhorrence of hydrodynamicists to the use of entropy because of its abstractness—and claims that thermohydrodynamics takes on a clear mathematical form only when entropy is given a leading role.

To prove his point that entropy is neglected, author cites five "standard" works whose indexes do not list "entropy." Average date of publication of these five works is 1937; if author consults the vast literature on gas dynamics of the past decade or two, ample evidence will be found that thermodynamics is in current use and that entropy is appreciated.

Reviewer agrees that fluid dynamicists two decades ago generally considered thermodynamics superfluous, but notes that today thermodynamical arguments share equal importance with dynamical ones in problems involving compressibility, heat conduction, and temperature-dependent viscosity. Reviewer questions author's broad assertion that the specific appearance of the entropy in the mathematical equations is highly desirable, and, to the author's coined words "entrophobe" and "entrophile," he would add "entropomane." A. H. Shapiro, USA

896. Kaye, J., Keenan, J. H., Brown, G. A., and Shoulberg, R. H., Heat-transfer measurements in an inexpensive supersonic wind tunnel. I. Apparatus and results for a laminar boundary layer based on a simple one-dimensional flow model, *J. appl. Mech.* **22,** 3, 289–296, Sept. 1955.

Paper describes experiments for supersonic flow of air in two $^1/_2$-in. diam tubes with length-to-diameter ratios of about 32 and 50. Diameter Reynolds numbers ranged from 20,000 to 492,000; length Reynolds numbers from 2.27×10^6; and tube inlet Mach numbers were 3.0 and 2.8. For one-dimensional flow model, the following results are plotted versus length Reynolds number for a series of duct Reynolds numbers: local Stanton number, local heat flux, local wall temperature, and modified ratio of static pressure to inlet total pressure. The local Stanton number is compared with theoretical predictions using methods of Van Driest [*NACA TN* 2597] for plate flow, and Toong [ScD. thesis, M.I.T., 1952] for tube flow. At low duct Reynolds number, data tend to agree with tube-flow predictions, while at higher duct Reynolds numbers data agree with plate predictions. Transition region from laminar to turbulent boundary layer is clearly apparent in all plots. Because of careful experimental techniques, data appear to reviewer to be very reliable. R. L. Mela, USA

897. Kaye, J., and Brown, G. A., Heat-transfer measurements in an inexpensive supersonic wind tunnel. II. Results for a laminar boundary layer based on a two-dimensional flow model, *J. appl. Mech.* **22,** 3, 297–304, Sept. 1955.

Data described in preceding review are analyzed with the aid of an approximate two-dimensional flow model (2-DFM), and a comparison is made with a one-dimensional flow model (1-DFM). Local values of Mach number, boundary-layer thickness, Nusselt number, and Stanton number, are plotted vs. length Reynolds number at various values of duct Reynolds number. Mach number remains fairly constant in 2-DFM, whereas it drops appreciably in one 1-DFM. Local Stanton number is decreased, but corresponding Reynolds number is increased, so that 2-DFM points fall on approximately the same line on the *St* versus *Re* plot. Stanton and Nusselt numbers are compared with tube and plate-flow theories. Trend from tube flow at low Reynolds numbers to plate flow at high Reynolds numbers persists (see preceding review). Authors offer possible explanations for this trend.

Reviewer feels excellent progress has been made on an ambitious and worth-while project and looks forward to appearance of future papers by authors concerning analytical phases of this program and further interpetation of data.

R. L. Mela, USA

898. Fultz, D., and Nakagawa, Y., Experiments on overstable thermal convection in mercury, *Proc. roy. Soc. Lond. (A)* **231,** 1185, 211–225, Aug. 1955.

A report covering a basic study in thermal convection in mercury. Apparatus and techniques are well described for a study of the thermal instability in rotating layers of mercury. Data acquired indicate that the instability is evidenced by the setting in of oscillations of increasing amplitude. The results are in agreement with the theoretical work of Chandrasekhar. The temperature gradients at which the thermal instability starts and the periods of the oscillations formed through experiment agree with theoretical predictions. In their concluding remarks, authors present suggestions as to how the experimental program may be extended and improved for the benefit of other investigators. G. A. Hawkins, USA

899. McLellan, C. H., An investigation of the melting of bodies due to aerodynamic heating, *Trans. ASME* **77,** 5, 727–733, July 1955.

A one-dimensional heat-transfer model is used to study the aerodynamic heating problem of long-range ballistic-type vehicles re-entering the earth's atmosphere. Both laminar and turbulent cases are considered. Computations for a re-entering speed of 20,200 fps show that the heat-transfer rates may be great enough to melt the nose of the vehicle and, in some cases, the sides. Effects of dissociation and radiation are neglected.

Wind-tunnel tests at a Mach number of 6.9 are also reported, using four different configurations of models made of Wood's metal. Photographs showing the melting of these models are included. T. Y. Toong, USA

900. Chu, B.-T., Pressure waves generated by addition of heat in gaseous medium, *NACA TN* 3411, 47 pp., June 1955.

Mathematical analysis of the pressure waves produced in a gas by the addition of heat. Two cases are considered: (1) Moderate rate of heat release. A linearized solution for the pressure field is obtained. Effect of heat conduction is neglected. (2) High rate of heat release. An exact solution is given when a finite amount of heat is released uniformly at a section of a tube at a constant rate (large or small). An analysis is also made for the pressure waves generated by a point source in three dimensions. Some applications of the theory to combustion problems are discussed. A. D. Yoffe, England

901. Mawardi, O. K., On acoustic boundary layer heating, *J. acoust. Soc. Amer.* **26,** 5, 726–731, Sept. 1954.

Author improves the classical solution of Kirchhoff for the motion of an acoustic wave in a circular tube by including the effect of viscous dissipation. In brief, the argument is that, although the particle velocity is small and permits linearization of the equation of motion, the dissipation function occurring in the energy equation contains terms dependent on the velocity gradient. In

the boundary layer on the wall of the tube, the velocity gradient is appreciable and cannot, therefore, be neglected in the linearization process. At the same time, author relaxes the Kirchhoff boundary condition of an isothermal wall and considers instead a tube of finite wall thickness whose outer surface is maintained at constant temperature.

An experiment is described in which the temperature was measured at the wall of a tube containing a resonating air column. Excellent agreement with the theory was found.

P. R. Owen, England

902. Toong, T.-Y., A new examination of the concepts of adiabatic-wall temperature and heat-transfer coefficient, *Proc. second U.S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., June 1955, 811–816.

Solutions of momentum, continuity, and energy equations for the case of steady two-dimensional flow of an incompressible fluid with zero pressure gradient have been made. Numerical solutions for the differential equation have been obtained by means of an IBM Card Programmed Calculator. The distribution of adiabatic-wall temperature is found for a flat plate insulated downstream of an initial isothermal zone on the wall. Also the variation in heat-transfer coefficient is obtained under the conditions that the wall temperature is given a step change from the adiabatic value. These solutions have been made for comparison with conditions actually occurring with experimental data under so-called adiabatic or isothermal conditions. Actually, a so-called adiabatic plate is difficult to maintain adiabatic in portions of the wall near the leading edge; and isothermal wall conditions are difficult to maintain up to the leading edge. The good agreement between the solutions which take into account these difficulties and experimental data is shown graphically.

D. M. Mason, USA

903. Mark, M., and Stephenson, M., The air-cooled electronic chassis, ASME Advance paper no. 55—A-55, 10 pp. + 11 figs., 1955.

A method of cooling certain types of electronic equipment incorporates in the chassis a simple plate-fin heat exchanger, sometimes called a cold plate or cooled chassis. To determine which internal configuration gives most efficient heat transfer, equations for cold plates are developed, and fin density, height, and thickness are varied. The effect on required power, coolant flow rate, and pressure drop is noted, and some conclusions drawn as to optimum design. From authors' summary

904. De Corso, S. M., and Coit, R. L., Measurement of total emissivities of gas-turbine combustor materials, *Trans. ASME* **77,** 8, 1189–1197, Nov. 1955.

The total emissivity of a number of metal and ceramic coatings is measured by comparing the radiation from the surface heated electrically to a known temperature determined by means of an embedded thermocouple with the radiation from a black body at the same temperature. When a ceramic coating is used, a correction is made by calculation, assuming a knowledge of the thermal conductivity of the coating. Reviewer considers this method to be susceptible to appreciable errors, although the results will be sufficient for the purpose required.

It is shown that the emissivity of a bare metal surface rises as the metal is heated, while a sandblasted surface has always a high emissivity. The use of a suitable ceramic coating can prevent excessive rise in emissivity with heating. Hence the use of a ceramic coating on the inside of the combustor wall and sandblasting on the outside can reduce the temperature of the wall very considerably. M. W. Thring, England

905. King, J. I. F., A potential-theory formulation of radiant-heat transfer, *Quart. J. roy. meteor. Soc.* **81,** 349, 414–417, July 1955.

The radiant-heat flux in a homogeneous atmosphere can be represented as the gradient of a suitably defined radiant-heat potential because it constitutes an irrotational vector field. This potential is shown to be a functional of the product of the radiant-source function at all points in the atmosphere and an attenuation factor to allow for the absorption of the intervening medium.

The formalism is independent of the choice of coordinate system. The familiar equations of radiant transfer, such as the Milne-Schwarzschild equilibrium equation, are deducible as special cases for the particular geometry of the plane-parallel atmosphere. Most importantly, the radiant heating at an arbitrary point is expressed as a functional of the second derivative of the Planck intensity. This radiant-heat-flow equation will form the basis for the construction of a new atmospheric radiation chart.

From author's summary by A. Whillier, South Africa

906. Ward, G. T., Performance of a flat-plate solar heat collector, Inst. mech. Engrs. Preprint, 16 pp.

Tests on a two-glass, water-heating solar collector were used to establish a performance equation for the special case where the entering water is at the same temperature as the outside air. From consideration of the unsteady state it was concluded that the greatest efficiency would result from a collector of small mass and small water content. The economic study, for Malaya, indicates that solar energy is competitive with local fuels as a source of low-temperature (less than 175 F) heat.

Reviewer believes that in the efficiency equation (24) the flow rate W in the denominator should be divided by the effective transmittance (for solar radiation) of the glass. Furthermore, it should be emphasized that the conclusion that efficiency is independent of the rate of insolation is valid only for a collector in which the entering water is at the same temperature as the outdoor air; it is not valid for solar collectors in general.

Although useful experimental data are presented, reviewer believes that the paper is of limited value because no method is given for deriving the constants in the performance equation from the fundamental properties of the collector components. Thus the output of any other design of collector cannot be predicted from the equations given without doing actual tests on the proposed new collector. In contrast to this, the equations of Hottel [*Trans. ASME*, pp. 91–104, 1942] are completely general and may be used for predicting the performance of any kind of solar collector. A. Whillier, South Africa

907. Burnell, J. G., The discharge of boiling water through a Venturi tube, *J. Instn. Engrs., Austral.* **27,** 7/8, 213–214, July/-Aug. 1955.

Test results are given for the flow of boiling water through a venturi tube and a nozzle with the back pressure raised above the critical. This work is an extension of that described by the author in his paper "The flow of boiling water through nozzles, orifices and pipes" [title source, Mar. 1946].

From author's summary

908. Westwater, J. W., and Santangelo, J. G., Photographic study of boiling, *Indust. Engng. Chem.* **47,** 8, 1605–1610, Aug. 1955.

Authors present a discussion of photographic equipment and techniques used in studies of boiling and an evaluation of the results obtained.

Steam-heated, horizontal, copper tube was the hot element and

methanol was the boiling liquid. Over-all temperature differences ΔT between steam and methanol varied from 64 to 300 F. Heat-transfer coefficients were measured for each over-all ΔT by metering the methanol condensate after boiling. Photographic equipment permitted still photographs at 10^{-6}-sec exposure and high-speed motion pictures at 4000 frames per sec.

Paper contains good description of the mechanisms involved in the three regions of boiling: nucleate ($\Delta T < 100$ F), transition ($100 \text{ F} < \Delta T < 140$ F), and film boiling ($\Delta T > 140$ F), with six still photographs corresponding to different conditions. Quantitative results are given on bubble growth, frequency of bubble emission, space between active nuclei, film oscillation, shape of rising bubbles, etc.

No theoretical considerations are given in this paper.

A. E. Brun, France

909. **Rauber, M., Natural circulation in steam boilers** (in French), *Chaleur et Industrie* **36**, 358, 143–156, May 1955.

Liquid-vapor circulation in boiler tubes is studied semi-empirically, using simple equations supplemented by experimental data. Two actual examples of faulty circulation are examined theoretically, and improvements are suggested.

N. Ream, England

910. **Lorenzini, R. A., The dual-circulation boiler in the industrial power plant,** ASME Ann. Meet., N. Y., Nov. 28–Dec. 3, 1954. Pap. 54—A-214, 16 pp.

911. **Andrews, H. I., Heat losses of locomotive boilers,** *Engineering* **180**, 4672, 209–215, Aug. 1955.

912. **Forster, H. K., On the conduction of heat into a growing vapor bubble,** *J. appl. Phys.* **25**, 8, 1067–1068, Aug. 1954.

Paper is an addendum to an earlier article [AMR **7**, Rev. 3361], explaining considerations there which led to the formulation of the temperature of a fast-growing spherical vapor bubble in an infinite superheated liquid. For the integration of Rayleigh's equation there is a need for knowing the temperature of the moving bubble wall. The problem has been dealt with in different ways. Author, however, shows a new approach, mainly consisting of, first, solving the stationary heat-conduction problem (constant bubble radius), and then by applying the influence of the radial velocity, which leads to a simple expression of the temperature of the moving bubble wall. The success of applying this expression is apparent from the paper cited above.

C. E. Lenngren, Sweden

913. **Walker, E. H., Eltringham, R. M., and Puttick, A., Evaporation losses from petrol storage tanks in the United Kingdom—a practical survey,** *J. Inst. Petrol.* **41**, 382, 297–325, Oct. 1955.

This paper presents thermal breathing loss measurements obtained from both standing and working tanks located in a number of places in the United Kingdom. A method is given by which the results can be correlated and used to predict thermal breathing losses from readily available data. The results emphasize the importance of maintaining the tank and its fittings in a vapor-tight condition. The mathematical basis of the survey is given; formulas used are derived, and their method of application explained.

From authors' summary

914. **Vérnon, M., and Trèves, F., Analytical theory of cooling towers** (in French), *C. R. Acad. Sci. Paris* **240**, 26, 2493–2497, June 1955.

Paper presents a theory of two-stage heat exchangers in which gases such as ammonia in pipes are condensed by water streaming down past them, the water in turn being cooled by air currents. The theory is crude, the discussion is brief, and no traceable references are given. It is stated at the end that further work is to be published, but no clue as to place is given.

K. Stewartson, England

915. **Van Tijen, H. W., A contribution to the calculation of surface condensers,** *Proc. k. Ned. Akad. Wet.* (*B*) **58**, 1, part I, 19–29; part II, 30–39; part III, 40–48, 1955.

A method is developed for predicting the performance of large steam condensers and the variation with cooling water velocity, cooling water temperature, and condenser loading. Basis of method is a "condenser coefficient" determined from geometry of condenser bundle and flow conditions in condensing region on steam side. Extensive data of van Hengel are used to develop method and check its application. Graphs and sample calculation illustrate use of method. Numerous simplifications are used in developing method which is only intended to apply to steam condensers operating within usual commercial range of operation. Also, method has been checked on only two condensers over a limited range of variables.

Reviewer believes that geometrical factors have been oversimplified and that additional data are needed to show worth and limitations of method.

O. P. Bergelin, USA

916. **Fleishman, B. A., Axially symmetric diffusion in a radial velocity field,** *Proc. second U. S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 777–780.

Introducing a relative "solute velocity" u defined by $u\rho = -\kappa\rho_r$ [ρ = concentration, κ = diffusivity], author solves the linear diffusion equation in a simple source flow, $U \sim r^{-1}$. He chooses boundary conditions amounting to $u = -U$ at the origin and at "semi-permeable" concentric boundaries which pass solvent but catch solute. Reviewer would have welcomed more detailed discussion of these boundary conditions.

S. Corrsin, USA

917. **Fritzsche, A. F., General appraisal of shell-and-tube heat exchangers with parallel and cross flow** (in German), *VDI-Forschungsheft* **21**, 450, 5–18, 1955.

An unusually excellent appraisal of factors affecting heat transfer in shell-and-tube heat exchangers. Method of dimensional analysis is used to fullest extent. Existing data provide most of general correlations presented. Unfortunately, the large amount of American manufacturers' data seems not to have been used. While article is comprehensive, much still remains to be done in correlating parameters suitable for evaluating performance of shell-and-tube heat exchangers. The dimensionless groups used are the customary Reynolds, Prandtl, and Nusselt forms. Some pressure drop data are also correlated, but the author seems unaware that he had been anticipated by the late A. P. Colburn. The relative insensitivity of the Prandtl group in some cases is rather startling.

The article remains by far the best on the subject to date.

J. M. DallaValle, USA

918. **Yoshida, U., and Suzuki, A., On the temperature measurement by the thermistor oscillator,** *Bull. Fac. Engng. Yokohama nat. Univ.* **4**, 57–62, Mar. 1955.

Paper reports upon the design of a temperature-measuring instrument. Thermistors were used as the resistive elements of an R-C oscillator, and the temperature change was detected by measuring the change in the output frequency. Temperatures were measured between 32–42 C and the accuracy was stated as ± 0.005 C with stabilized input voltage. Connection

diagrams of the oscillator, discriminator, and of the complete thermometer are given. P. Biringer, Canada

Combustion

(See also Revs. 760, 793, 795, 808, 812, 814, 904, 952)

919. Campbell, E. S., A method for integrating a set of ordinary differential equations subject to a type of numerical indeterminacy, *Univ. Wisconsin nav. Res. Lab.; Dept. of Chem.* (8) CM-847, 55 pp., July 1955.

Paper describes scheme for solving sets of ordinary differential equations useful when (1) equations are coupled in an appropriate manner, and (2) usual procedures fail because of loss of significant figures due to subtraction. Technique is applied to integrate equations for free-radical flame model. Report is amply illustrated with examples. Y. Luke, USA

920. Skirrow, G., and Wolfhard, H. G., Studies of chlorine trifluoride flames, *Proc. roy. Soc. Lond.* (*A*)**232,** 1188, 78–87, Oct. 1955.

Diffusion flames are described in which the fuels, hydrogen, carbon monoxide, carbon disulphide, water, ammonia, hydrocarbons, and organic halides are burned with chlorine trifluoride. Chemical equilibrium is almost complete in the main reaction zone as in oxygen diffusion flames, but the chlorine trifluoride flames possess several unusual features. Carbon formation is very pronounced with hydrocarbon fuels. Radicals such as C_2 and CN have a large partial pressure in mixed hydrogen + hydrocarbon/chlorine trifluoride flames, and their presence in a high-temperature region enhances the luminosity of the flames. It is suggested that the persistence of these radicals may be due to the very rapid diffusion of hydrogen into the main reaction zone, which effectively prevents the penetration of fluorine to the fuel side of the flame. Chlorine trifluoride flames are briefly compared with fluorine flames.

From authors' summary

921. Swarts, D. E., and Frank, C. E., Effect of hydrocarbon structure on reaction processes leading to spontaneous ignition, *NACA TN* 3384, 23 pp., July 1955.

Previous work showed that heptane and isooctane not only differed greatly in respect to their susceptibilities to vapor-phase oxidation but that the nature of the oxidation products was also different. These facts were interpreted on the basis of the temperature required for oxidative attack and on the thermal stability of the alkyl and peroxy radicals obtained. This work has been extended to other aliphatic hydrocarbons in the present investigation.

Experiments with 2,2,5-trimethylhexane, which differs from isooctane in having one more methylene group, show that this compound occupies an intermediary position between isooctane and *n*-heptane in susceptibility to oxidation. The addition of a methylene group reduces the resistivity of a hydrocarbon to oxidation.

The 2,2-dimethylbutane, which contains two secondary C—H bonds, is more easily oxidized than its isomer, 2,3-dimethylbutane, which contains two tertiary C—H bonds. This apparent anomaly may be due to the fact that tertiary C—H bonds in 2,3-dimethylbutane are sterically shielded from oxygen attack by the adjacent methyl groups.

In the temperature range from 350 to 450 C, 3-heptene gave oxidation products similar to those from *n*-heptane but showed greater resistance to oxidative attack. At temperatures of 500 to 550 C, isobutene appeared slightly more susceptible to oxidation than isobutane. Possible explanations for these observations are suggested.

When a reaction tube with $^1/_4$-in. inside diameter was used, isooctane gave oxidation products similar to those previously reported with the $^1/_{16}$-in. tube. However, for a given contact time the extent of reaction and the danger of explosion were much greater in the larger diameter tube. These results illustrate the importance of a high surface-to-volume ratio in controlling vapor-phase oxidation processes.

From authors' summary

922. Wagner, P., Burning velocities of various premixed turbulent propane flames on open burners, *NACA TN* 3575, 32 pp., Oct. 1955.

Using direct photographs of near-stoichiometric premixed propane flames at room temperature and pressure, author measured turbulent burning velocities for 3-fold range of burner diameters and 3.5-fold range of velocities. Variations of viscosity and density were obtained by substitution of helium or argon for nitrogen of air. Over Reynolds numbers involved, 2100 to 26,000, pipe-induced turbulence intensities were 4.1%, 2%, and 4.9% for mixtures containing N_2, He, and A, respectively. Results show ratio of turbulent to laminar burning velocity correlated by N_{Re}, but dependence of ratio on N_{Re} is different for mixtures with different diluents and does not vary with burner diameter as predicted. Better correlation is obtained with N_{Re} based on turbulence intensity at burner exit. W. P. Jensen, USA

923. von Kármán, T., and Penner, S. S., Fundamental approach to laminar flame propagation, Colloquium, AGARD, Cambridge Univ., Dec. 7–11, 1953. 36 pp.

This is an excellent and very complete article which develops a complete system of equations for the theory of laminar flame propagation which takes into account both heat conduction and diffusion. The eigenvalue problem determining the flame velocity is formulated. Two specific examples are given to compare theory with data.

The examples chosen are the decomposition of hydrazine and the decomposition of ozone. The burning velocities calculated in the equations developed agree closely with those measured by Lewis and von Elbe. Numerous assumptions are necessary for the numerical solution of the equation, but the results obtained are particularly good for small or moderate burning velocities. The influence of diffusion coefficients has been expressed by utilizing approximations. It is felt that all burning processes can be described by assuming that total enthalpy is constant. Further, burning velocities are calculated in systems of chain reactions involving the steady-state approximation. The steady state used here is in contradiction to that used by Lewis and von Elbe.

J. H. Davidson, USA

924. Olsen, H. L., and Gayhart, E. L., Incipient flame propagation in a turbulent stream, *Jet Propulsion* **25,** 6, 276–283, June 1955.

An excellent comparison of hot-air and flame kernels produced by spark ignition in laminar and turbulent flow. High-speed flash schlieren photography permits observations in a time range of 0–10,000μ sec.

Propane-air mixtures were used. Effects of variations in stream velocity, fuel-air ratio, turbulence grids, and spacing of spark relative to grid were studied.

Isotropic turbulence did not enhance flame propagation within the times used. Granularity of turbulent flames was strikingly similar for different scales of turbulence, suggesting it is a function of an inherent instability in flame induced by turbulence.

Nonisotropic turbulence does enhance apparent flame speed. For rich mixtures, flame speeds were lower, and it was possible to demonstrate that turbulence can result in flame-quenching.

Comparison of hot-air and flame kernels indicates effect of turbulence is partly due to turbulent diffusion or transport of burning material. R. C. Anderson, USA

925. **Bolz, R. E., and Burlage, H., Jr., The influence of turbulence on flame propagation rates,** *Jet Propulsion* **25,** 6, 265–275, 283, June 1955.

Flame speeds were determined from shadowgraphs, obtained with a high-speed motion picture camera, of a flame globule ignited by a single spark discharge in a free jet of fuel-air mixture. This is basically the same as the technique developed by H. L. Olsen and co-workers [cf. Olsen and Gayhart, AMR 6, Rev. 1370]. Technique gives observations of the transient phase of turbulence effect.

A natural gas fuel was used. Laminar flame speeds were in reasonable agreement with other measurements.

Measurements were made with different screens to give different scales of turbulence. Attempted correlation with an empirical equation was successful for only one screen. No correlation was obtained for quite lean or rich mixtures, since turbulent flame speeds were less than laminar.

R. C. Anderson, USA

926. **Gross, R. A., Boundary layer flame stabilization,** *Jet Propulsion* **25,** 6, 288–290, 293, June 1955.

Flat and airfoil-shaped plates were used as flameholders in premixed propane-air streams in laminar flow. Author observed two classes of flame stabilization—that in laminar boundary layer and that effected by an eddy formed by flow separation behind flame front.

For flat plates, author obtained blow-off gradients six times larger than found by Lewis and von Elbe, attributed to use of secondary air surrounding primary air, with both streams at equal velocity. Maximum free-stream velocity increased with plate length, as expected.

For airfoil sections, the shorter the section, the greater the free-stream velocity needed for blow-off. Author obtained oscillating conditions at low velocities with approximately stoichiometric mixtures. With manipulation flame front could be carried toward bow of airfoil and even held there momentarily. Temperature of airfoil above 600 F is required for oscillations.

D. Aronson, USA

927. **Schaffer, A., and Cambel, A. B., The effect of an opposing jet on flame stability,** *Jet Propulsion* **25,** 6, 284–287, June 1955.

As alternative to physical bodies as flame stabilizers, authors propose opposing gas jets. Tests were made with a jet discharging from a $^1/_{16}$-in. ID round tube in line, but opposed in direction, to main fuel-air stream in 1.77-in. ID Vycor tube. Mass flow of stabilizing jet was kept in range of 1% to 1.6% of main stream flow. Broader range of operation was obtained with jet-stabilized flame than with physical body as stabilizer.

Jet stream was operated at 100 psig except in one set of experiments to observe effect on flame length. Stabilization was obtained at all pressures. With change of gas in jet stream, operation was sharply affected. Nitrogen did not stabilize flame, whereas oxygen proved much better than air.

Stabilization is attributed to formation of a critical zone of high recirculation.

Reviewer considers this study to be an extension of long-established art of using impinging jets to stabilize a flame in can-type combustors for gas turbines. Such combustors operate with jets at low pressure ratio of 1.05 to 1.10, compared to ratios studied by authors of 3.0 to 7.0. D. Aronson, USA

928. **Cheng, S.-I., Low-frequency combustion stability of liquid rocket motor with different nozzles,** *Jet Propulsion* **25,** 4, 163–167, Apr. 1955.

This article investigates the effect of unsteady flow in the supercritical nozzle on the stability of the low-frequency pressure oscillation. The effect of the deviation of nozzle flow from being quasi-steady on the stability of low-frequency oscillation in liquid propellant rockets is investigated. Both monopropellant and bipropellant rockets are treated and the physical aspects of the results are discussed.

Characteristic equations are developed for neutral oscillations for the monopropellant case and mass balance equations for the bipropellant case.

It is shown for simple nozzles that about nine tenths of the converging portion of the nozzle volume should be added to the chamber volume to average gas residence time for the monopropellant rocket. However, in the bipropellant rocket, dissipation of engine from low-frequency disturbance is secondary compared to the volumetric effect.

This article is highly specialized but gives 10 references dealing with combustion stability and liquid rocket motors.

J. H. Davidson, USA

929. **Putnam, A. A., and Dennis, W. R., Burner oscillations of the gauze-tone type,** *J. acoust. Soc. Amer.* **26,** 5, 716–725, Sept. 1954.

The occurrence of vibrations in a flame burning from a grid in an 8-in. diam chamber was studied. The resulting acoustic oscillations are, perhaps, related to Rijke gauze tones. Self-excitation is discussed in terms of a phase lag between changes in flow rate and changes in heat-transfer rate. This satisfactorily correlates the experimental observations, but authors stress that the theory requires further detailed examination.

J. Barr, Scotland

930. **Markstein, G. H., and Squire, W., On the stability of a plane flame front in oscillating flow,** *J. acoust. Soc. Amer.* **27,** 3, 416–424, May 1955.

Recent experimental studies revealed a close connection between spontaneous excitation of organ-pipe oscillations by flames propagating in tubes and flame-front instability characterized by cellular structure. On the basis of these observations a dominant mechanism of vibratory flame movement is proposed, consisting of the following two steps:

(1) As a consequence of acceleration instability, the oscillation of the gas column alternatingly distorts and flattens the flame front, thus creating the observed vibration-induced cell structure.

(2) The distortions of the flame front cause in turn periodic changes of flame surface area, and, therefore, of burning rate. Spontaneous oscillations are excited if the phase relation between burning rate variation and pressure oscillation is favorable.

In this paper only the first step is analyzed. The linearized treatment yields stability limits in terms of dimensionless parameters representing amplitude and frequency of gas column oscillation and wave length of flame-front distortion. The properties of the flame front entering into the analysis are the ratio of densities of unburned and burned gases, the burning velocity of the plane flame front, and a characteristic length that determines its stability in the absence of oscillations. Results in qualitative agreement with experimental observations are derived.

From authors' summary by W. R. Dennis, USA

931. Weinberg, F. J., Location of the "schlieren image" in a flame, *Fuel* **34,** S84–S88, Apr. 1955.

Calculation is made of the location of the "schlieren image" in a unidimensional case. The position in a flame at which maximum deflection of an incident ray of light occurs is calculated in terms of temperature. It is concluded that this is situated at a temperature equal to twice the initial temperature (in degrees K) in a system of constant thermal conductivity and at one and a half times the initial temperature in air. Calculated temperatures are compared with the experimental results of several workers. E. C. Wilkerson, USA

932. Gaydon, A. G., Use of spectroscopy in elucidating reaction mechanism, Colloquium, AGARD, Cambridge Univ., Dec. 7–11, 1953, 12 pp.

This is an excellent review article on stressing the need for quantitative spectroscopic work. Emission spectra are studied, and it is established that hydroxal bands of a maximum intensity lie near stoichiometric point, whereas the maximum for CH bands lies close to fairly rich mixtures, and for C_2 bands close to even richer mixtures. Low-pressure flames are studied as well as atomic flames and flames in a fluorine atmosphere. Flash photolysis and activation energy are also discussed, as well as recent work on radical formation. Forty-eight references are supplied which bring the material up to date.
J. H. Davidson, USA

933. Grosvenor, G., Atomization of liquid fuels by high pressure natural gas, *Combustion* **27,** 4, 51–53, Oct. 1955.

To improve flame effectiveness in open-hearth steel making, studies were made of flame patterns of oil and gas burners and the influence exerted upon them by the inner cone with particular emphasis on the role of the atomizing agents—air, steam, and natural gas—the last of which produced particularly good results.
From author's summary

934. Chernov, N. N., Study of the distribution of gas flow in blast furnaces by means of models (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 1, 80–91, Jan. 1954.

A $^1/_{25}$ scale-model blast furnace was used to study distribution of gas flow. Measurements were made of volumes of gas passing in three concentric sections of furnace, viz: up the axis by the walls, and in the intervening annulus. The size grading of the charge was shown to have greatest effect on flow distribution across a section of the furnace. Tuyère size and pressure of blast played secondary roles. Static pressure at various points on a radius of the furnace were also measured and found to be most influenced by the degree of fragmentation of the charge.
J. K. Kilham, England

935. Baker, D. I., Mixture ratio and temperature surveys of ammonia-oxygen, rocket motor combustion chambers, *Jet Propulsion* **25,** 5, 217–226, May 1955.

936. Silver, R. S., and Mackay, R. W., Mass transfer theories of fuel bed combustion, *Brit. J. appl. Phys.* **6,** 8, 267–271, Aug. 1955.

Recently there has been considerable interest in the possibilities of developing theories of combustion based on mass-transfer ideas. The purpose of the present paper is to clarify the general position reached as a result of previously published work, particularly in relation to combustion of carbon in fuel beds. The authors chose, for their discussion, the published three theories of Thring, Silver, and Spalding, because each uses a different approach and is amenable to the process of intercomparison. From the present work, it is suggested that, in this way, an improved appreciation of the general theory will facilitate further advance and, in terms of this basic general theory, a method of evaluating an accurate transport parameter to describe a live fuel bed is available as a logical beginning for further progress. M. Kunugi, Japan

937. Flodin, C. R., and Haaland, H. H., Some factors affecting fly-ash collector performance on large pulverized fuel-fired boilers, ASME Ann. Meet., N. Y., Nov. 28–Dec. 3, 1954. Pap. 54—A-212, 12 pp.

Acoustics

(See also Revs. 750, 781, 808, 855, 901, 929)

938. Herrey, E. M. J., Experimental studies on acoustic radiation pressure, *J. acoust. Soc. Amer.* **27,** 5, 891–896, Sept. 1955.

Author describes a device similar to the radiation balance which can be used to measure the normal and shearing components of the force on a disk arising from the radiation pressure. The results obtained agree with Brillouin's contention that the radiation pressure arises from a unidirectional stress in the sound beam. Author suggests that the same apparatus can be used for measuring the absolute power output of transducers and for measuring the average reflection transmission and absorption coefficients of disk-shaped samples as a function of angle of incidence. Results for cork, aluminum, and steel plates are reported. R. D. Spence, USA

939. Henrici, P., Application of two methods of numerical analysis, to the computation of the reflected radiation of a point source, *J. Wash. Acad. Sci.* **45,** 38–45, 1955.

The problem is to evaluate numerically the double integral $\int_0^\infty dt \int_0^{2\pi} tu^{-3}v^{-2}d\phi$, where $u^2 = 1 + \xi^2 + t^2 - 2t\cos\phi$ and $v = t^2 + \eta^2$. The integral is first transformed into a single integral involving a hypergeometric function. The hypergeometric function is calculated from its power-series expansion and the integral evaluated numerically. Most of the paper is concerned with estimates of the errors produced by truncation of the power series and by the quadrature. In order to speed up the convergence of the series, Aitken's δ^2-method is studied in detail. For the quadrature errors, complex variable methods recently developed by Davis and Rabinowitz [*Math. Tables Aids Compt.* 8, 193–205, 1954] are employed. It is reported that values of the original integral for the two ranges $\xi = 0.5(.05)1.6$, $\eta = .05(.05)1.6$ and $\xi = .25(.25)8.0$, $\eta = .25(.25)8.0$ have been calculated and are kept on file.
Courtesy of Mathematical Reviews P. W. Ketchum, USA

940. Knudsen, J. R., The effects of viscosity and heat conductivity on the transmission of plane sound pulses, *J. acoust. Soc. Amer.* **26,** 1, 51–57, Jan. 1954.

Author considers analytically the problem of discontinuous change in the flow of gas in a semi-infinite tube and includes the linear terms representing the effects of viscosity and thermal conductivity in the equations. He examines the pulse which is caused to propagate down a tube when (1) a constant flow of gas is suddenly introduced into the tube in which gas is at rest, (2) a constant flow of gas is suddenly started and later suddenly stopped. These pulses have profiles which are initially (1) a step function and (2) a rectangular function, respectively. Profile (1) is shown to propagate with a steepness which decreases as the time after launching increases and the second steady state, i.e. constant flow along the tube, is approached as $t^{-3/2} \to 0$. Profile

(2) is shown to propagate as a Gaussian function whose amplitude decreases as $t^{-1/2}$ while its width increases as $t^{1/2}$. There is a negative velocity of flow in the wake of this pulse which tends to zero as $t^{-3/2} \to 0$. The distortion of the profile and the time of decay are increased by large coefficients of viscosity and conductivity in each case.

The calculations reported are for the ratio of (*conductivity/viscosity*) = *ratio of specific heats*, but qualitatively the findings remain valid for most values of conductivity and viscosity.

M. J. P. Musgrave, England

941. **Garrick, I. E., and Watkins, C. E., A theoretical study of the effect of forward speed on the free-space sound-pressure field around propellers,** *NACA Rep.* 1198, 16 pp., 1954.

See AMR **7,** Rev. 1992.

942. **Litovitz, T. A., and Carnevale, E. H., Effect of pressure on sound propagation in water,** *J. appl. Phys.* **26,** 7, 816–820, July 1955.

Measurements of the ultrasonic absorption of water have been made up to pressures of 2000 kg/cm². The data show that both the shear and compressional losses decrease as the pressure is raised. Comparison of these results with the values predicted by Hall's theory of sound absorption in water shows that the theory must be modified to be in agreement with the experimental data. The modification consists of assuming that the open or ice type of packing is associated with higher free-energy state. If this modification is made, the observed data are in excellent agreement with theory. This result leads to the conclusion that, even at 0 C, less than 30% of the molecules in water are arranged in the ice type of packing. The results also indicate that the free-energy difference between the two states of packing in water is temperature dependent.

From authors' summary

943. **Tatarskii, V. I., On the theory of the propagation of sound waves in a turbulent stream** (in Russian), *Zh. eksp. teor. Fiz.* **25,** 1 (7), 74–83, 1954 (translated from Russian by M. D. Friedman, 572 California St., Newtonville, Mass., 15 pp.).

Author considers the problem of sound dispersion by isotropic turbulence in an incompressible fluid and utilizes the classical technique of solving the wave equation. Some of the assumptions are scientifically naive; e.g.: "Since the scalar random temperature field does not correlate with the velocity field in an incompressible fluid, these two effects can be considered independently. The general effect of dispersion will be simply the sum of the considered effects." The work of J. E. Moyal of Great Britain has shown that while the field variables separate into two physically distinct groups they are, nevertheless, coupled through the nonlinear inertia terms in the dynamic equations of the fluid. Furthermore, the work of M. G. Odintsov and I. G. Shaposhnikov of USSR introduces the fact that the effect of turbulence on acoustic wavé propagation depends on its scale. While it is true that the author considers the question of an incompressible fluid, the implications of the paper seem to be less general than is apparently intended. L. Liebermann of the United States indicated that the refractive effects of the inhomogeneities result in local warping of the wave fronts, thereby producing intensity augmentation or diminution at the receiver, when the fluid being considered is incompressible.

The validity of the author's treatment of his subject is satisfactory, but its applicability is restricted to idealized cases despite the use of perturbation techniques to increase the generality of the proposed theory.

Reviewer believes that the theory propounded in this paper is not in agreement with the influence of scale of turbulence on the propagation equations as demonstrated by M. G. Odintsov and I. G. Shaposhnikov (USSR) in 1949.

N. M. Matusewicz, USA

944. **Gerrard, J. H., Measurements by the sound from circular cylinders in an air stream,** *Proc. phys. Soc. Lond.* (*B*) 68, part 7, 427b, 453–461, July 1955.

Periodic vortex shedding accounts for the tones observed when a gas flows past a rigid cylinder with Reynolds numbers in the range 50 to 10^5. The frequency and intensity of the resulting sound were observed by placing suitable cylinders in a wind tunnel or by whirling them in stationary air. The distribution of the intensity is that of a dipole source, and the value of the intensity varies with Reynolds number in a manner similar to that of the ratio (*pressure drop at rear of cylinder*)/(*velocity head*).

V. Salmon, USA

945. **Callaghan, E. E., and Coles, W. D., Investigation of jet-engine noise reduction by screens located transversely across the jet,** *NACA TN* 3452, 27 pp., May 1955.

An investigation of screens located transversely across a jet as a noise-reduction device was conducted on a full-scale turbojet engine. The screens lowered the sound pressure levels rearward and increased them in front, resulting in a nearly circular nondirectional sound field. The sound power level was lowered by as much as 7.5 db by the proper screen choice and location, and, in addition, the maximum sound pressure level was decreased approximately 12 db over that of the engine alone.

From authors' summary

946. **Snow, W. B., Instrumentation for noise measurements,** *Noise Control* **1,** 3, 16–21, 56, May 1955.

The various modern instruments for measuring noise are described and illustrated, and their purpose and range of application are outlined. From author's summary

947. **Beranek, L. L., Labate, S., and Ingard, U., Acoustical treatment for the NACA 8 × 6-foot supersonic propulsion wind tunnel,** *NACA TN* 3378, 86 pp., June 1955.

Report summarizes the results of a project at the Lewis Flight Propulsion Laboratory to silence the 8- by 6-ft supersonic wind tunnel. Sound measurements in the neighborhood surrounding the tunnel were conducted to evaluate the noise-attenuation requirements. A muffler-development program was continued until these attenuations were achieved. The final design for the acoustic treatment is described and experimental performance curves are compared with anticipated theoretical results.

From authors' summary

948. **Michelson, I., Theory of vortex whistle,** *J. acoust. Soc. Amer.* **27,** 5, 930–931, Sept. 1955.

The characteristic feature of B. Vonnegut's vortex whistle is the proportionality of pitch to flow rate [B. Vonnegut, title source, **26,** p. 18, 1954], in contrast with more common types of whistles in which geometry determines the pitch. A formula for the performance of vortex whistles was proposed by its discoverer on the basis of tests; it is confirmed on the basis of elementary physical considerations which show that high velocity and multidimensionality of the jet are essential. From author's summary

949. **Ścibor-Marchocki, R. I., Analysis of Hypex horns,** *J. acoust. Soc. Amer.* **27,** 5, 939–946, Sept. 1955.

Impedance transformations of the "Hypex" horns are derived for all cases of mismatch for both increasing and decreasing

characteristic impedance. A method of using a Smith chart for these transformations is presented. The results are applicable to cases where losses are appreciable. From author's summary

950. Marcu, J., Design and calculation of turbo pumps and compressors approaching cavitation and sound velocity limits (in German), *Maschinenbau-Technik* **4**, 6, 313–324, June 1955.

These seemingly different but actually somewhat similar limits of the two machines particularly influence the inlet diameter and inlet vane angle of the impeller. With major simplifying assumptions these are figured and shown in diagrams as functions of other characteristic parameters for optimum design. The article, translated from Rumanian, is an abstract of the section treating the same subject of two German books of Pfleiderer: "Kreiselpumpen für Flüssigkeiten und Gase" (Ref 6) and "Strömungmaschinen," Springer 1952. It uses the same explanations, equations, and figures without adding anything new. The Rumanian text is useful for his countrymen, but the translation in a German periodical is hard to justify. A. Hollander, USA

Ballistics, Detonics (Explosions)

(See also Revs. 638, 808, 814, 899, 928)

951. Winter, M. R., Ballistic homologies for variable site in the real atmosphere (in French), *Mém. Artill. fr.* **29**, 111, 7–71, 1955.

St. Robert (as did Galileo earlier) considered the motion of a projectile as determined by the drop due to gravity from the uniform rectilinear motion occurring (at the muzzle) at the instant of departure. This analysis ignores air resistance; but in a modified form, allowing for air resistance, but only along the line of departure, it serves reasonably well to predict terminal conditions for small variations in angle of departure matched against small changes in angle of site of target.

Present author, using the laws of air resistance of the GHM ballistic tables, constructs tables and graphs of correction multipliers so that, in modified form, St. Robert's scheme for differential corrections in range and time of flight, due to change in angle of elevation, can be extended accurately. No other kind of homology is here considered. Geometric and kinematic effects are examined. A graphical method of analysis is also developed.
A. A. Bennett, USA

952. Carlton, C. H., Warren, F. A., Wiegand, J. H., The industrial use of high-energy materials, *Chem. Engng. Progr.* **51**, 7, 335–338, July 1955.

Many substances, alone or as mixtures, can be placed in the category of high energy materials. Of these, explosives, propellants, and pyrotechnics form a group whose wide utilization by the Armed Forces is fully appreciated but whose use by industry may not be so fully realized. This paper reviews the industrial use of these materials and discusses possible future applications.
From authors' summary by A. O. Schmidt, USA

953. Ludwig, G., Oscillation of rockets upon re-entering the atmosphere (in German), *Weltraumfahrt* **5**, 2, 43–48, Apr. 1954.

Methods resolving problems associated with rocket oscillations upon missile re-entrance in the atmosphere are both timely and important. Author treats an elementary case of interest which neglects the coupling of the oscillations between the tangent to the path and those of the longitudinal axis, during re-entry. The assumed physical model is described by an equation of the form $\ddot{\alpha} + d(t)\dot{\alpha} + c(t)\alpha = 0$ where α, a function of time, is the angle of attack, $d(t)\dot{\alpha}$ and $c(t)\alpha$ are damping and restoring, respectively. A distinction is made for the cases in which (1) $c(t)$ is still small so that during one half oscillation, $c(t)$ and $d(t)$ have a large percentage change; (2) $c(t)$ is so big that, during one half oscillation, $c(t)$ has only a small percentage change. A standard transformation is made which leads to an expression for the solution as a Bessel function. To facilitate numerical work, a number of numerical approximations are made. Next, author considers the motion in a plane, taking into account the coupling between oscillations of the longitudinal axis of the missile and the tangent to the path. The motion is described by two equations, one of which gives the oscillations about the center of gravity, the other the unperturbed trajectory of the center of gravity. A solution is then given based on approximations for numerical work.

Reviewer notes that the approximations are introduced in order to reduce the tedious computing otherwise required. The numerical approximations can, however, be valid only in certain regions and these are not adequately described. It would have been very desirable for the author to illustrate his procedures by carrying through one representative calculation. It is unfortunate that no mention or consideration is given to solving the system by means of a high-speed digital computer. The equations lend themselves very well to solution by available electronic machines. Although the paper is well formulated and quite understandable, the work is plagued with typographical errors.
J. W. Fischbach, USA

Soil Mechanics, Seepage

954. Lubinski, A., The theory of elasticity for porous bodies displaying a strong pore structure, *Proc. second U.S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 247–256.

For elastic, homogeneous, isotropic, porous, and permeable media, in the case of steady-state flow, author examines laws of elasticity, assuming that both porosity and permeability are independent of pressure. Problems may be solved without knowledge of pore shape. The method uses a special stress-strain relationship and makes allowance for body forces due to flow through the medium and for permeable and impermeable boundaries. The method is equivalent to superimposing a hydrostatic stress, a stress similar to thermal stress, and a stress resulting from subjecting the body, assumed nonporous, to pressure.

For transient flow conditions, the findings are valid only for porous media displaying a strong pore structure, by which it is meant that pressure distribution at any time is known, in so far that it may be calculated by other methods. As an example, stresses in long circular cylinders subjected to radial flow are investigated. From author's summary by Á. Kézdi, Hungary

955. Caudle, R. D., and Clark, G. B., Stresses around mine openings in some simple geologic structures, *Univ. Ill. Engng. Exp. Sta. Bull.* **430**, 41 pp. May, 1955.

956. Blekhman, I. I., Examining the process of vibratory driving of piles and sheet piles (in Russian), *Inzhener. Sbornik, Akad. Nauk SSSR* **19**, 55–64, 1954.

Author deals analytically with the influence of the soil, pile, and vibrating driver parameters on the effect of driving or extracting. The soil resistance is considered as the total of the frictional and point resistance in the ordinary way without any hypothesis of some essential changes of physical properties of soil due to the vibration effect. The general equations resulting on this basis, and taking into account both the elastic and plastic resistance.

agree well with the experimental statements and explain them. By neglecting the elastic part of the total resistance, author gives final solutions for the calculation of the maximum depth, average speed, and time of driving or extracting. Diagrams, facilitating the calculation, are given. L. Šuklje, Yugoslavia

957. **Aronofsky, J. S., and Ferris, O. D., Transient flow of non-ideal gases in porous solids—one-dimensional case,** *J. appl. Phys.* **25,** 10, 1289–1293, Oct. 1954.

Flow equations are derived in finite difference form for the case of sudden reduction of pressure at one end of a finite one-dimensional tube of porous material. The gas in the tube differs from an ideal gas in the viscosity and compressibility factor, both of which are represented by simple functions of pressure. Numerical solutions of equations were obtained on an electronic computer for various combinations of viscosity and compressibility factor. Results are presented in small graphs and show that pressure decline and production history of the system are influenced by variations in these parameters. W. D. Baines, Canada

Geophysics, Meteorology, Oceanography

(See also Revs. 839, 840, 841, 871, 955, 957)

958. **Küssner, H. G., Theory of the elastic-plastic continuum with free surface under variable loading** (in German), *ZAMM* **35,** 4, 130–143, Apr. 1955.

In geophysics the problem arises of finding displacements in a semi-infinite gravitational body subjected to forces varying arbitrarily with time. Paper presents an approach which assumes homogeneity and isotropy without otherwise restricting material properties. Small displacements of the continuum can be described as products of linear integral operators representing the external loads, and of characteristic functions satisfying the remaining boundary conditions.

After a two-page discussion of elastomechanical, hydrodynamic, and thermodynamic theory of homogeneous continua, which covers viscoelastic as well as elastic-plastic materials, particular solutions for small velocities are considered. In the remaining sections are discussed boundary conditions at the free surface, loads moving on the surface, bending of an elastic plate floating on the free surface under static load, and bending of an elastic plate under a load imposed at time $t = 0$. Some tables giving numerical values of deflections under loads on strips and circular plates, and sinking velocities for the circular plates, are included in the last section. The exact physical problems to which these tabulated values correspond, and the units in which they are given, are not at all clear to reviewer.

A. D. Topping, USA

959. **Press, H., and Meadows, M. T., A reevaluation of gust-load statistics for applications in spectral calculations,** *NACA TN* 3540, 19 pp., Aug. 1955.

Authors analyzed airplane measurements of atmospheric turbulence by generalized harmonic or spectral methods.

The spectra for the vertical component of turbulence appear to be inversely proportional to the square of the frequency argument. This spectral curve agrees reasonably with the shape suggested by the theory of isotropic turbulence for a given frequency range. However, a flattening of the spectrum is indicated for lower frequencies.

Authors base their work on the following assumptions:

(1) The turbulent velocity fluctuations for a given weather situation, as well as the airplane response for a linear system, have a normal probability distribution.

(2) The form of the power spectrum of the gust velocity is given by

$$\Phi(\Omega) = \sigma_w^2(L/\pi)(1 + 3\Omega^2L^2)/(1 + \Omega^2L^2)^2$$

where $\Omega = 2\pi/\lambda$ and has the dimensions of radians per ft, σ_w^2 is the mean square gust velocity, and L is the scale of turbulence.

By using two additional asumptions, that the accelerations were due entirely to vertical gusts and that the airplane was assumed to respond in vertical motion only (no pitch-rigid body condition), authors were able to develop a series of equations which enabled them to relate the probability distribution of the root-mean-square gust velocity to the over-all number of peak accelerations per second. Thus, available data could be reevaluated without recourse to the original records.

Some of the figures indicate the variation of the probability distribution of the root-mean-square gust velocity: (1) for a transcontinental and a local feeder-line operation; (2) with altitude; and (3) with the three weather conditions, clear air turbulence, cumulus clouds, and thunderstorms.

Reviewer hopes that further research will be conducted into extending the present analytical techniques or developing newer ones so as to more completely account for the aircraft dynamics, in order to relate gust-load spectra to the appropriate meteorological variables. H. Lake, USA

960. **Rounds, W., Jr., Solutions of the two-dimensional diffusion equations,** *Trans. Amer. geophys. Un.* **36,** 3, 395–405, June 1955.

Author gives general solutions of the two-dimensional diffusion equation applicable to flow over a flat surface with and without an upper boundary. Diffusion of gases and aerosols with zero free fall velocity and particles with nonzero free fall velocity are considered. Several different sources (infinite in cross-wind extent) and located at different heights above the lower surface are considered. The down-wind concentrations evaluated from the different initial conditions are shown in graphical form. Application to diffusion under an atmospheric inversion is suggested.

R. O. Reid, USA

961. **Gifford, F., Jr., Atmospheric diffusion from volume sources,** *J. Meteor.* **12,** 3, 245–251, June 1955.

Paper is an extension of Frankiel's solution of problem of diffusion from instantaneous spherical volume due to isotropic turbulence. Paper derives, by virtue of its generality, limiting solutions of Sutton and Roberts respecting point sources. Several special cases are developed and compared, with results in general of smaller magnitude than expected from point sources. A compact, useful, and readable paper. J. M. DallaValle, USA

962. **Bunker, A. F., Turbulence and shearing stresses measured over the North Atlantic ocean by an airplane-acceleration technique,** *J. Meteor.* **12,** 5, 445–455, Oct. 1955.

A method of measuring the horizontal and vertical turbulent components of the wind has been developed to meet the need for these measurements over the open ocean. An anemometer, a vertical accelerometer, and a gyroscope mounted in an airplane (U. S. Navy PBY-6A) yield simultaneous records of the total airspeed, vertical accelerations, and the attitude of the airplane. From these records, and from airplane-lift theory and the characteristics of the particular type of aircraft, the vertical and horizontal fluctuations of the wind, the root-mean-square deviation velocities, and the shearing stress can be computed. The method allows the phugoid oscillations of the airplane to contribute nearly periodic velocity terms to both components; these must be eliminated by a computational step. A different reduction

method can give phugoid-free vertical velocities, but an additional instrument will be required for phugoid-free horizontal velocities. The airplane lift has a response of from 80 to 100% to gusts of radius 20 m or more, and the method fails to measure reliably gusts larger than 350 m. This limits the contributions to the quantities presented to those by gusts within the 20- to 350-m range. In the lower levels of polar-continental air masses, root-mean-square vertical velocities have been observed to attain 191 cm/sec over land and 120 cm/sec over water, while stresses reached maxima of 16.6 dyne/cm² over land and 3.8 dyne/cm² over water. Shearing stresses in the lowest 60 m in the trade winds averaged 0.2 dyne/cm² over seven days, while vertical velocities averaged 26 cm/sec. The influence of atmospheric stability and convective activity upon the turbulent velocities is discussed.

From author's summary

Lubrication; Bearings; Wear

(See Rev. 767)

Marine Engineering Problems

(See also Revs. 666, 784)

963. Van Manen, J. D., Thrust deduction and a proposed formula for its radial distribution, *Inter. Shipbldg. Progr.* **2,** 8, 153–166, 1955.

Problem concerns the design of ship screws for the condition of minimum energy loss of the system composed of ship + screw. First is discussed the flow phenomena behind the ship and the influence of the screw on these. The possible contributions of the local flow, the wave system, and friction are considered. A critical discussion is given of the conclusions of Dickmann, Van Lammeren, and Weitbrecht on the contribution of the frictional effects.

The relationship of local frictional thrust deduction and the local frictional wake fraction is carefully explored for two cases: a radially unequal velocity field (at the ship's stern) and a velocity field both radially and circumferentially unequal. Finally, a formula is deduced for the distribution of the radial thrust deduction coefficient, and is applied to four cases with quite satisfactory results.

This paper is an important one in the field of design of wake-adapted screws for minimum energy loss and will bear careful reading. B. W. Augenstein, USA

964. Timman, R., and Vossers, G., On the Michell's expression for the velocity potential of the flow around a ship, *Inter. Shipbldg. Progr.* **2,** 6, 96–102, 1955.

The velocity potential of the flow around a slender ship is derived by Fourier integral transform. The solution is obtained by the condition that the disturbance vanishes at infinity ahead of the ship, instead of the artificial device made by Havelock and Michell. The equivalency of Havelock and Michell's expression is proved. M. Kataoka, Japan

965. Hogben, N., A new integrator for use in wave-resistance calculations, *Trans. Instn. nav. Arch. Lond.* **96,** 3, 167–181, July 1954.

A satisfactory theoretical approach to the problem of wave resistance is an urgent need. At present, although a mathematical solution is known, it is laborious to apply and, due to the assumptions made, is not easily correlated with experiment. The integrator described in this paper is designed to lighten the computing burden which is encumbering research in this field. Its construction is not complicated, and a reasonably efficient and accurate prototype has been made without great difficulty. It has the virtue that it performs continuously integrations which, in general, must be carried out by approximate finite difference methods, to which they do not always readily lend themselves. Operation is so simple that a wave-resistance calculation, normally an arduous piece of computing, is reduced to an almost trivial routine. Detailed instructions for performing such a calculation are given in this paper, together with certain relevant charts and tables and a worked example. It is further shown how the instrument can, if desired, be used to compute Fourier coefficients and may thus assist in work such as the study of strength and vibration problems, where harmonic analysis is much used. This account is also illustrated by an example. Such an instrument should certainly be a valuable research tool.

From author's summary by W. C. Johnson, Jr., USA

966. Nordström, H. F., Edstrand H., and Lindgren, H., On the influence of form upon skin friction resistance, *Medd. SkeppsProvAnst. Göteborg* no. 31, 28 pp., 1954.

Paper describes experimental work carried out by authors to further explore the effect of form on resistance of ship hulls. Frictional resistance of ship hulls is normally computed by assuming wetted area of hull can be treated as a flat plate of equal area and flat-plate friction coefficients applied. It has long been known that some error is involved in this procedure; however, it has been considered accurate enough to treat form resistance as part of residual resistance and expand it from model to ship by Froude's law of comparison.

Authors have examined effect of breadth of hull upon skin friction by testing models of two prismatic coefficients, 0.80 and 0.60, and additional models of these same prismatic coefficients, block coefficients, and principal dimensions except that beam was successively changed to $^1/_2$ and $^1/_4$ of that of original models. Also two double models, i.e., symmetrical about the water plane, were tested submerged. Further tests were made with one model in which draft was varied.

Results of resistance tests were plotted against Shoenherr's friction line. Resistance coefficient differences in fully turbulent flow range below wave-making speeds were lifted from the curves and plotted against displacement-length ratio. Results of previous tests of earlier models made by authors, together with data obtained by other experimenters, are also shown. Curve drawn through the points shows a resistance coefficient difference varying from about 0 at $\Delta^{1/3}/L = 0.10$ to 20% at $\Delta^{1/3}/L = 0.18$. Certain of the points varied widely from the mean curve.

Although data from authors' experiments have no immediate practical value, they shed more light on effect of form on ship hull resistance and are of considerable interest to the ship model experimeter. R. B. Couch, USA

967. Duncan, J. P., Experimental solutions of propeller shaft vibration problems, *J. Instn. Engrs., Austral.* **27,** 3, 57–66, Mar. 1955.

Two torsional vibration problems encountered in shipboard propeller-engine combinations are discussed from an elementary point of view. The thesis developed is the obvious one that experimental techniques and electrical analogs are valuable in solving mechanical design problems. W. P. Targoff, USA

Biomechanics

(See Rev. 887)